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E. Eseller-Bayat,[1] *S. Gokyer,*[2] *M. K. Yegian,*[3] *R. O. Deniz,*[4] *and A. Alshawabkeh*[3]

Bender Elements and Bending Disks for Measurement of Shear and Compression Wave Velocities in Large Fully and Partially Saturated Sand Specimens

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ABSTRACT: Bender element and bending disk setups were designed and implemented in a liquefaction box that accommodated a large sand specimen (190 mm × 300 mm × 490 mm). The box was manufactured for testing fully and partially saturated sand specimens under cyclic and earthquake excitation. Special housings for bender elements (31.8 mm × 12.7 mm × 0.51 mm) were manufactured to allow insertion of the elements through the side walls of the box. The bender elements were used to measure shear wave velocities in multiple directions to ascertain the uniformity of a sand specimen prepared in the liquefaction box. Large bending disks (31.8 mm × 0.41 mm) in special housings were utilized to measure compression wave velocities with the aim of investigating the presence and effect of the degree of partial saturation in specimens. The housings for bender elements and bending disks were designed to minimize the boundary effects of the Plexiglas walls of the liquefaction box. This paper provides details of the experimental setup and the designs of the bender element and bending disk housings. Sample test results of shear and compression wave measurements are also included to demonstrate successful application of the test setup. The experimental setup described is well suited for utilizing bender elements and bending disks for shear and compression wave velocity measurements in fully and partially saturated large sand specimens.

KEYWORDS: bender element, bending disk, shear wave velocity, S-wave velocity, compression wave velocity, P-wave velocity, piezoelectric transducer, piezoceramics, partially saturated sand, liquefaction box, large sand specimens

Introduction

One of the innovative non-destructive techniques used in geotechnical experimental testing utilizes piezoelectric transducers made of piezoelectric ceramics, typically in the form of bender elements and bending disks. The most common application of piezoelectric transducers is as bender elements in many different test setups such as triaxial, oedometer, resonant column, shake table, and centrifuge tests to assess sample disturbance and estimate wave velocities (Shirley and Hampton 1978; Horn 1981; Schulthesis 1981; Dyvik and Madshus 1985; Thomann and Hryciw 1990; Agarwal and Ishibashi 1991; Gohl and Finn 1991; Viggiani and Atkinson 1995; Brignoli et al. 1996; Zeng and Ni 1998; Lee and Santamarina 2005; Leong et al. 2005; Landon et al. 2007; Hoyos et al. 2008; Chen et al. 2008; Fu et al. 2009; Kim and Kim 2010; Zhou et al. 2010). In most of the reported applications, the soil samples were small (up to 165 mm), and measurements of wave velocities were made using small size transducers that generated little electric energy but large enough waves to be detected at the receiver. There are few studies (Kim and Kim 2010) in which tip-to-tip distances are as large as 300 mm. For compression wave measurements, bender elements have also been used by Yang (2002), Tsukamoto et al. (2002), Tamura et al. (2002), Ishihara and Tsukamoto (2004), and Naesgaard et al. (2007) in small triaxial specimens (sample size up to 156 mm) and by Lee and Santamarina (2005) in oedometer tests (tip-to-tip distance of up to 150 mm).

The authors have been involved in research aimed at developing a new liquefaction mitigation measure that is based on inducing partial saturation in otherwise fully saturated loose sands. The initial phase of the research was to develop an experimental test setup that would permit the preparation of fully and partially saturated large sand specimens in a special liquefaction box and testing of the specimens on a shaking table to assess the beneficial effect of partial saturation on the liquefaction potential of the sand. For this purpose, it was important to ascertain whether (1) the specimen preparation technique followed in the laboratory would indeed yield samples with uniform relative density (D_r) and (2) the degree of partial saturation can be determined through measurement of the compression wave velocity.

Piezoelectric transducers (bender elements and bending disks) were used in conjunction with the liquefaction box to achieve the above-stated objectives of the research. The application of these

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[1] Asistant Professor, Istanbul Technical Univ., Istanbul 34469, Turkey.

[2] Ph.D. Candidate, Northeastern Univ., Boston, MA 02115, United States of America.

[3] Professor, Northeastern Univ., Boston, MA 02115, United States of America.

[4] QC Head of Shift, Treviicos Corporation, Jamestown, KY 42629, United States of America.

transducers was quite challenging because of the large size of the soil specimens that were tested, the three-dimensional wave forms generated in the liquefaction box, and the fixed boundary effects on wave transmission. To accommodate the measurement of wave velocities at large distances (up to 270 mm), large transducers, a power amplifier, and a high resolution digital oscilloscope were used. To ascertain the uniformity of the sand specimen in the liquefaction box, multiple large bender elements (31.8 mm × 12.7 mm × 0.51 mm) were used. To evaluate the effect of partial saturation on the compression wave velocity, large bending disks (31.8 mm × 0.41 mm) were used instead of bender elements, thus avoiding potential interference problems between compression and shear wave velocities in partially saturated sands as reported by Lee and Santamarina (2005). Special housings for both bender elements and bending disks were manufactured that minimize the boundary effects of the Plexiglas walls.

This paper presents the details of the liquefaction box, the housings of bender elements and bending disks, and the overall test setup for measuring shear and compression wave velocities of large sand specimens in three dimensions. Sample test results are presented to demonstrate the successful application of the experimental setup developed.

Piezoelectric Transducer Setup

Overview of Experimental Setup

Figure 1 shows a photograph of the experimental setup that was used in the application of piezoelectric transducers for wave velocity measurements. The figure shows the liquefaction box resting on the shaking table in which fully and partially saturated sands were prepared. Also included in Fig. 1 is the measurement setup consisting of a signal generator, a power amplifier, and a digital oscilloscope (Yokogawa DL750 Scopecorder with 12 digital channels for recording and signal processing) that were used for transmitting, receiving, and recording shear and compression waves using bender elements and bending disks, respectively.

The liquefaction box was designed and manufactured to accommodate multiple bender elements and bending disks, which were installed in the box through its four side walls and at two elevations. Figure 2 shows a drawing of the liquefaction box in which the locations of the piezoelectric transducers are identified.

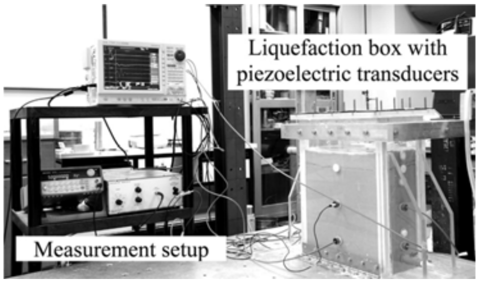


FIG. 1—*Experimental setup used in the application of piezoelectric transducers for shear and compression wave velocity measurements.*

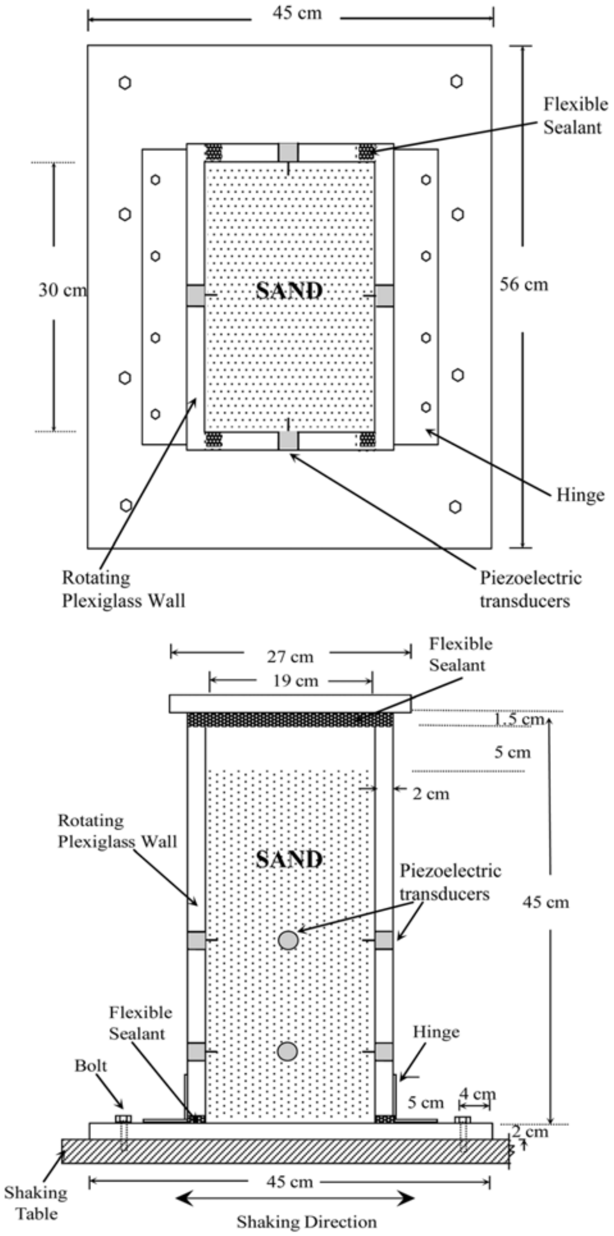


FIG. 2—*Plan and elevation sections of the liquefaction box.*

Design and Manufacturing of Piezoelectric Transducers

Bender elements and bending disks were used to measure shear and compression wave velocities, respectively, in fully and partially saturated sand specimens. A summary of their designs, manufacturing, and installations in the liquefaction box follows. Further specific details have been presented by Deniz (2008).

Bender Elements

The selection of the size and type of the bender element was based on a number of considerations, including the distance the wave traveled in the soil specimen, the energy of the wave that needed to be generated in order for it to be accurately detected at the

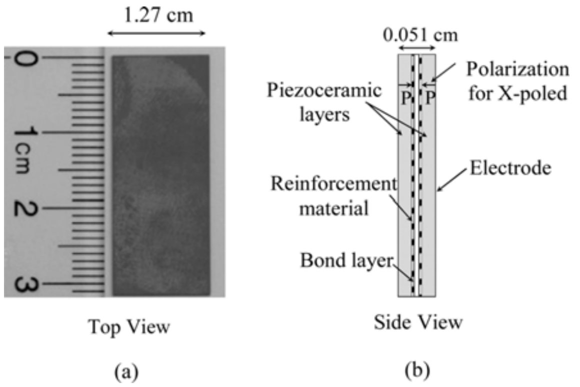


FIG. 3—*(a) Bender element used (PSI-5A4E type). (b) Typical configuration of a bender element (horizontal scale is magnified 10×).*

receiver, and the ease of manufacturing and mounting the element in the liquefaction box. Figure 3 shows the picture of the bender element (PSI-5A4E) used in this research as a transmitter as well as a receiver. The bender element was 31.8 mm long, 12.7 mm wide, and 0.51 mm thick and made of lead zirconate titanate. The bender elements used were primarily X-poled.

The insertion of the bender elements into the sand specimens through the walls of the liquefaction box required the construction of a special housing. Figure 4 shows the housing, which consists of a 19.2 mm diameter brass pipe with male threads at one end to be mounted into the Plexiglas wall of the liquefaction box with bender elements centered to ensure proper alignment between the transmitting and receiving elements. The inside of the brass fitting was then filled with Devcon epoxy to fix the bender element in the housing. Details of the design and manufacturing of the bender elements, including the wiring of electric connections, waterproofing, housing into soil specimens through the side walls of the liquefaction box and grounding can be found in one of the co-authors' M.S. thesis (Deniz 2008).

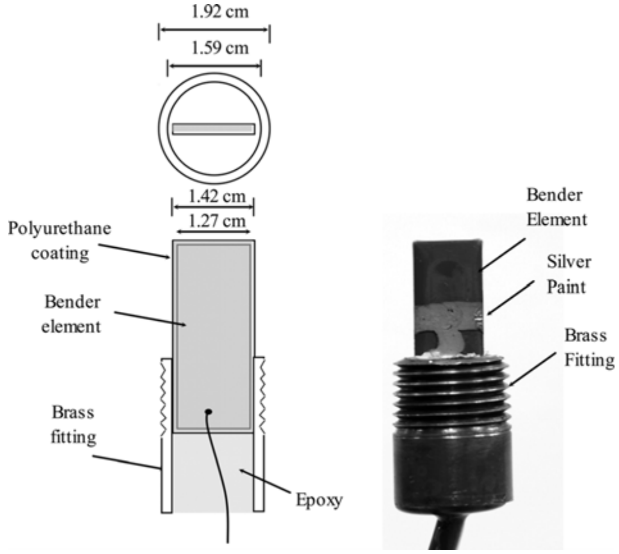


FIG. 4—*A sketch and a photograph of a bender element with its housing.*

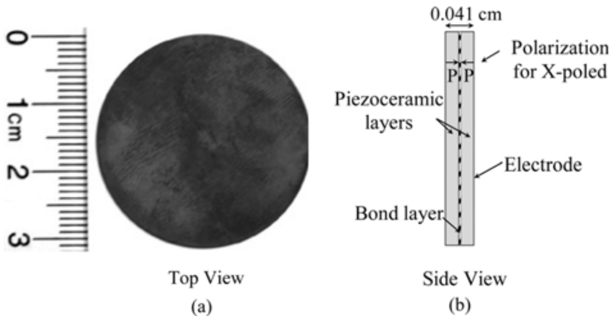


FIG. 5—*(a) Bending disk used (PSI-5A4E type). (b) Typical configuration of a bending disk (horizontal scale is magnified 10×).*

Bending Disks

The selection of the type and size of the bending disk was primarily based on its capacity to provide a high enough amplitude of transmitted compression wave and consideration of its ease of insertion in the liquefaction box. The selected bending disk was PSI 5A4E type with a diameter of 31.8 mm and a thickness of 0.41 mm as shown in Fig. 5. The bending disks used were X-poled.

Similar to bender elements, the design and manufacturing of the bending disks included the wiring of electric connections, waterproofing, housing into soil specimens through the side walls of the liquefaction box, and grounding.

The housing for a bending disk was designed and manufactured to accommodate its large size and required perimeter restraint to allow the bowing action. The housing consisted of a hollow polyvinyl chloride (PVC) bushing with male thread at one end to allow its insertion from inside the liquefaction box into the female threaded holes on the Plexiglas side walls. Figure 6 shows a cross section and photographs of a bending disk with its housing. The bending disk was attached around its perimeter to the wall of the bushing with silicone. The flexibility of the silicone provided the added benefit of preventing high frequency vibrations generated by the disk from being transmitted through the Plexiglas box to the receiver. As noted in Fig. 6, holes were made along the side of the PVC bushing to allow soil and water to fill inside the housing during specimen preparation. This was essential to ensure balanced soil pressures on both sides of the disk and to have proper coupling with the surrounding soil.

Example Test Results and Interpretation

Selected test results are presented to demonstrate the application of the bender element and bending disk setup in the measurement of wave velocities in fully and partially saturated large sand specimens. Shear and compression wave measurements were made to confirm the uniformity of the density of the specimens, as well as to evaluate the effect of partial saturation on wave velocities.

Sample Preparation

Eight bender elements (for shear wave measurements) or bending disks (for compression wave measurements) in their housings were inserted through the four Plexiglas side walls of the box at

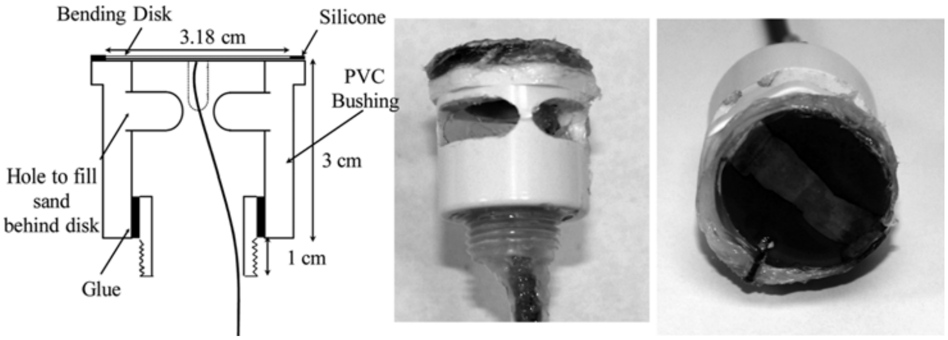
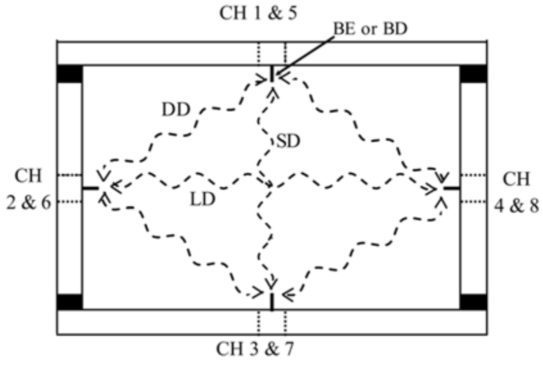


FIG. 6—*Sketch and photographs of a bending disk with its housing.*

two elevations (Fig. 2) prior to the preparation of sand specimens in the liquefaction box. Fully saturated sand specimens were prepared by wet pluviation of dry Ottawa sand (ASTM graded C778) rained into the liquefaction box, which was partially filled with water. The sand surrounding the bender elements and bending disks was gently tapped to ensure proper contact between the elements and the sand. The density and the degree of saturation of the fully saturated specimen were computed using phase relationships with measurements of the amount of water, the dry mass of the sand, and the height of the specimen, which represents the total volume of the specimen. Partially saturated sand specimens were prepared again by wet pluviation, but this time the dry Ottawa sand was mixed with powdered sodium perborate monohydrate. The chemical reaction of the powder with water generated minute gas bubbles. Therefore, wet pluviation of the Ottawa sand mixed with powdered sodium perborate monohydrate enabled gas generation in the pore space of the sand, inducing a partial degree of saturation. The degree of saturation was controlled by the amount of powdered chemical mixed with dry Ottawa sand.



Long Distance, LD = 27 cm (BE), 23 cm (BD)
Short Distance, SD = 15.5 cm (BE),10 cm (BD)
Diagonal Distance, DD = 15.6 cm (BE), 9.5 cm (BD)

CH 1-4, Lower Level (33 cm)
CH 5-8, Upper Level (22 cm)

FIG. 7—*Locations of bender elements (BE) and bending disks (BD) and their signal recording channels (CH).*

Shear Wave Velocity Measurements

The bender element setup was used to evaluate the uniformity of the density of the sand specimens prepared following the procedure described above. Shear wave velocity measurements were made along short and long directions, as well as at upper and lower levels, 220 mm and 330 mm from the surface of the specimen in the box. Figure 7 shows the numbering of the bender elements and their channels, as well as the distances between the elements measured tip to tip.

Figure 8 shows a screen shot of the typical oscilloscope output from a shear wave velocity test showing a single pulse transmitted wave (bender element [BE] at channel [CH] 2) and received waves at three other bender elements (CHs 1, 3, and 4). As expected, the waves arrived at BEs 1 and 3, which are the same distance from the transmitter; at BE 4, which is a longer distance from the transmitter, the wave arrived at a later time.

Figures 9(*a*) and 9(*b*) present plots of transmitted and received signals in a partially saturated sand specimen ($S = 77\,\%$) with a relative density (D_r) of 21 % along long and short distances, respectively. The measurements were made at a depth of 330 mm, corresponding to an effective stress (σ'_v) of 3 kPa. This effective stress was calculated with phase relations using the height of the sand and the height of the water and with knowledge of the amount of water and sand used to prepare the specimen. As gas bubbles were generated in the voids, they replaced the volume of water. The volume of water replaced by gas bubbles was

FIG. 8—*Screen shot of the oscilloscope from a typical shear wave velocity measurement.*

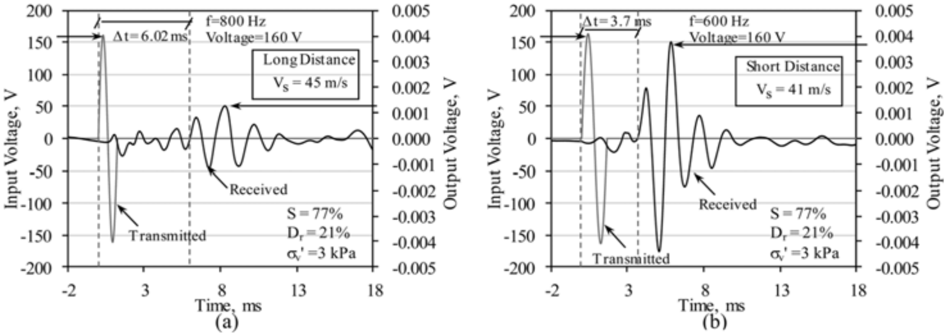


FIG. 9—*A comparison of measured shear wave velocities in a partially saturated sand specimen in the long and short distances (LD and SD, respectively).*

evaluated by measuring free water that accumulated on the specimen surface. The frequencies indicated on the plots correspond to the highest wave amplitude generated and detected, which typically was close to the resonant frequency of the bender element in the tested specimen. The shear wave velocities along the long distance (BEs 2 and 4) and short distance (BEs 1 and 3) were calculated as 45 m/s and 41 m/s, respectively, as shown in Figs. 9(*a*) and 9(*b*). The wave velocities were computed using a first arrival time approach (Viggiani and Atkinson 1995; Brignoli et al. 1996; Leong et al. 2005; Lee and Santamarina 2005; Landon et al. 2007). The close agreement between the velocities (45 and 41 m/s in Fig. 9) in the long and short directions within the specimen confirms the uniformity of the density of the specimen prepared by wet pluviation of Ottawa sand in the liquefaction box.

Figure 10 shows computed shear wave velocities from tests run on a fully saturated sand specimen with a similar relative density ($D_{\text{r}} = 21\,\%$) but at two different vertical effective stresses ($\sigma'_{\text{v}} = 3$ kPa and $\sigma'_{\text{v}} = 17.4$ kPa). A vertical effective stress of 17.4 kPa was achieved by stacking lead plates on top of the sand specimen in the size of the surface of the specimen. The computed shear wave velocities shown in Figs. 10(*a*) and 10(*b*) are for wave velocities between bender elements 1 and 3 (short distance). The shear wave velocities under the two effective stresses ($\sigma'_{\text{v}} = 3$ kPa and $\sigma'_{\text{v}} = 17.4$ kPa) were 52.5 m/s and 77.5 m/s, respectively. The increase in the shear wave velocity due to an increase in the effective stress can be verified using the empirical shear wave velocity formulation of Seed et al. (1986) for granular materials. In this formulation, the shear wave velocity for granular material with a given relative density (for a constant $(\text{N}_1)_{60}$ value) is proportional to the fourth root of σ'_{v}. Thus, a predicted value of the shear wave velocity under 17.4 kPa would be equal to $52.5 * (17.4/3)^{0.25} = 81.3$ m/s, which is close to the measured value of 77.5 m/s. However, it is recognized that the effective stress range tested was relatively low.

A comparison of computed shear wave velocities in fully ($S = 100\,\%$) and partially ($S = 77\,\%$) saturated sand specimens with similar relative density (D_{r}) and vertical effective stress ($\sigma'_{\text{v}} = 9.6$ kPa) also achieved by lead weights is presented in Fig. 11. Shear wave velocities for $S = 100\,\%$ and $S = 77\,\%$ are 69.8 m/s and 66.8 m/s, respectively. The close agreement between the shear wave velocities of the fully and partially saturated sand specimens with similar initial relative densities confirmed that the partial saturation did not change the vertical effective stress conditions.

Figure 12 shows computed shear wave velocity measurements in loose ($D_{\text{r}} = 21\,\%$) and dense ($D_{\text{r}} = 70\,\%$) fully saturated sand specimens under the same vertical effective stress, ($\sigma'_{\text{v}} = 3$ kPa). As expected, the shear wave velocity of the dense sand (70 m/s) is greater than that of the loose sand (52.5 m/s). Again, using Seed et al.'s (1986) formulation, the predicted value of the shear wave velocity of the dense sand based on the value of the loose sand is calculated as 67.3 m/s, which is close to the measured value of 70 m/s.

In summary, the selected test results presented and their interpretation demonstrate that the bender element test setup can accurately measure the shear wave velocities in large sand specimens and thus can be used to confirm the uniformity of the density of sand specimens, as well as to evaluate the effect of various soil

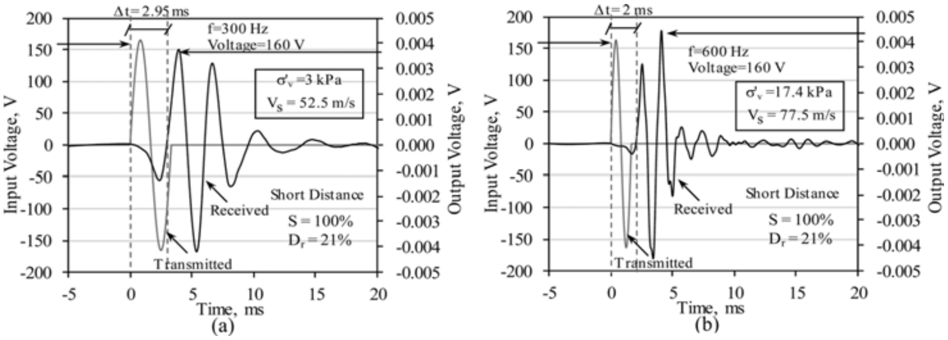


FIG. 10—*A comparison of shear wave velocity results in a fully saturated sand specimen at different vertical effective stresses.*

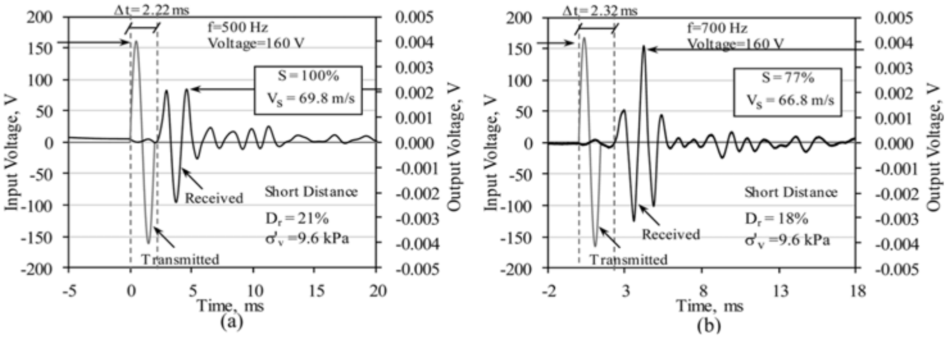


FIG. 11—*A comparison of shear wave velocity results in fully and partially saturated sand specimens.*

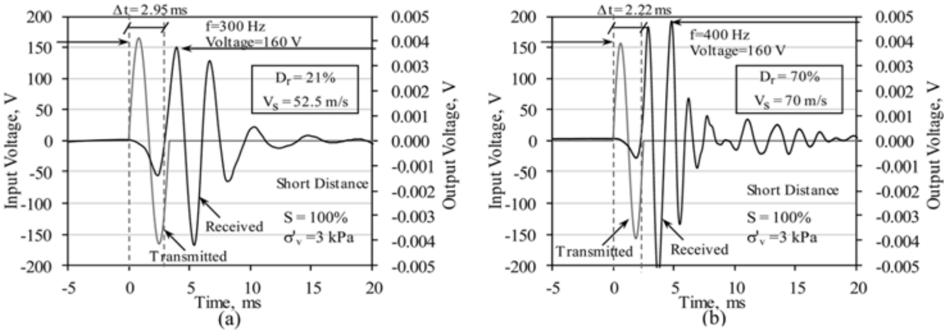


FIG. 12—*A comparison of shear wave velocity results in fully saturated sand specimens with different relative densities.*

parameters such as the degree of saturation and the relative density on the shear wave velocity.

Compression Wave Velocity Measurements

Tests were conducted in the liquefaction box in fully and partially saturated sands using bending disks to determine the effect of partial saturation on the compression wave velocity (V_p) and to investigate whether the degree of partial saturation can be indirectly determined through the measurement of V_p. The relative densities of the samples were in the range of 20 % to 30 %. Eight bending disks with their housings were inserted into a sand specimen through the four side walls of the liquefaction box at the same locations as described for the bender elements in Fig. 7. To detect compression wave arrivals more accurately, high frequency (5–15 kHz) single pulse sinusoidal transmitted signals were determined to be most suitable. Proper grounding was ensured, and stacking of 32–256 signals with the help of a function generator was applied to minimize the effect of high frequency noise on the arrival of compression waves. The digital oscilloscope recorded all the signals and then averaged the data. Because the noise is an ambient signal, averaging weakens the effect of the noise on the arriving signal and can recover a weak hidden wave associated with the compression wave.

Figure 13 shows a typical computed V_p in a fully saturated sand specimen. As expected, the computed $V_p = 1568$ m/s is close to the V_p of water. Figure 14 shows a typical compression waveform in a partially saturated sand specimen with a degree of saturation $S = 82$ %. As expected, because of the compressibility of

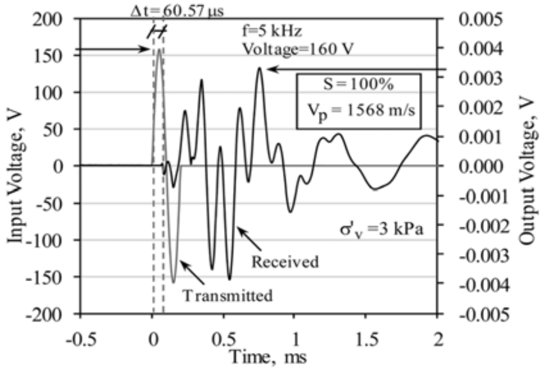


FIG. 13—*Measured compression wave velocity V_p in a fully saturated sand specimen.*

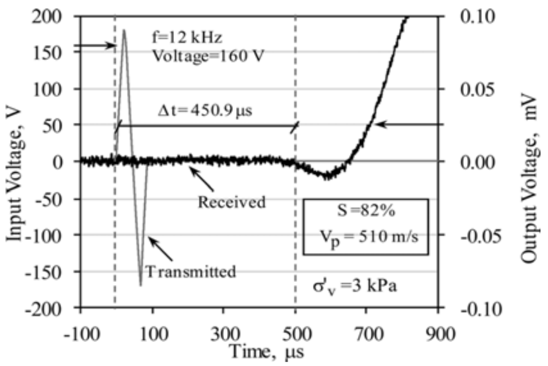


FIG. 14—*Measured compression wave velocity V_p in a partially saturated sand specimen with $S = 82$%.*

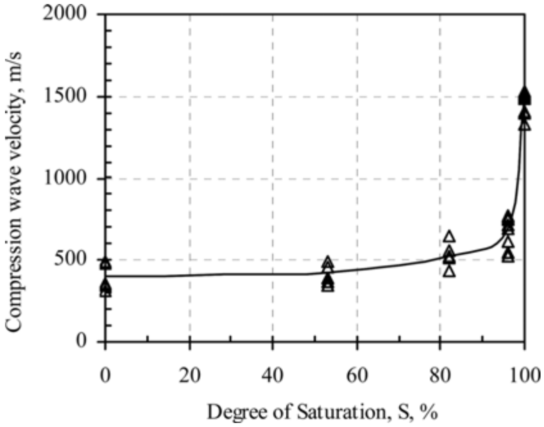


FIG. 15—*Compression wave velocity V_p measured using bending disks as a function of the degree of saturation.*

the pore water/gas in the partially saturated specimen, the V_p (510 m/s) is smaller than that of a fully saturated specimen (1568 m/s).

Figure 15 shows the computed V_p values as a function of the degree of saturation S. It is noted that a very slight reduction in the degree of saturation (from $S = 100\,\%$ to $S = 96\,\%$) leads to a dramatic decrease in V_p (on average from 1493 to 680 m/s), whereas a significant decrease in the degree of saturation from $S = 96\,\%$ to $S = 0\,\%$ (dry sand) slightly reduces V_p on an average from 680 to 398 m/s. This slight decrease in V_p is within the experimental uncertainty of ±100 m/s. Therefore, while a computed V_p value of less than 1500 m/s indicates partial saturation, a value of less than 1500 m/s cannot be accurately correlated to the degree of saturation S.

Summary and Conclusions

The measurement of shear and compression wave velocities in large fully and partially saturated sand specimens (up to 270 mm) prepared in a liquefaction box required special designs and the construction of bender elements and bending disks with their housings.

The bender elements were utilized to evaluate the uniformity of the density of the sand specimens tested. Bending disks were used to investigate the effect of partial saturation on the compression wave velocity. Special housings for both bender elements and bending disks were manufactured that minimized the fixed boundary effects from the Plexiglas walls of the liquefaction box. Sample test results are included that demonstrate the successful application of the experimental test setup.

Acknowledgments

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