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## Design and Application of Simple Shear Liquefaction Box

**REFERENCE:** Eseller-Bayat, E., Gokyer, S., Yegian, M. K., Ortakci, E., and Alshawabkeh, A., "Design and Application of Simple Shear Liquefaction Box," *Geotechnical Testing Journal*, Vol. 36, No. 3, 2013, pp. 1–9, doi:10.1520/GTJ20120025. ISSN 0149-6115.

**ABSTRACT:** A cyclic simple shear liquefaction box (CSSLB) was designed and manufactured to allow laboratory testing of saturated sands under cyclic or transient strain-controlled conditions. The box accommodates pore water pressure transducers, linear variable displacement transducers, and bender elements and bending disks. To induce shear strains, the box has two rotating walls connected to two translating rigid walls with a flexible sealant. When the tops of the rotating walls are fixed against translation and the base of the CSSLB, which rests on a shaking table, is excited with a displacement time history, shear strains are induced in a soil specimen. Two-dimensional numerical analyses of plan and elevation sections of the CSSLB were performed, demonstrating that the design of the box and the mechanism for shearing can induce controlled shear strains in sand specimens with minimal boundary effects. Example test results from sand specimens subjected to cyclic and earthquake shear strain time histories are presented and illustrate how well the CSSLB with its instrumentation is suited for conducting tests on relatively large soil specimens.

**KEYWORDS:** liquefaction box, liquefaction, shear strain, FLAC, boundary effects, shaking table, cyclic shear strain, transient shear strain

### Introduction

Research aimed at understanding the behavior of liquefiable sands continues to be an important focus of earthquake engineers. Often, laboratory tests are conducted to evaluate the cyclic behavior of sands. For this purpose, laboratory triaxial or simple shear tests are typically performed (Roscoe 1953; Bjerrum and Landva 1966; Moussa 1974; Budhu, 1985). Large-scale shake table and centrifuge tests are also used to understand the behavior of sands under seismic excitations (Dobry and Abdoun 2001; Thevanayagam et al. 2009; Dashti et al. 2010). There is a need for a simple test setup that can be used to investigate the behavior of relatively large laboratory sand specimens under cyclic or 1 *g* seismic excitations.

The authors have been involved in research aimed at developing a liquefaction mitigation measure that is based on introducing gas bubbles in fully saturated sands to reduce the degree of saturation. They have demonstrated that a minute amount of small gas bubbles entrapped within the pores of a loose sand will significantly reduce, if not eliminate, the potential for liquefaction (Yegian et al. 2007). This research included preparing fully and

partially saturated sand specimens and testing them under cyclic shear strains. The primary purpose of such tests was to demonstrate the benefit of partial saturation in reducing excess pore water pressure generation in sands subjected to shear strains. A special cyclic simple shear liquefaction box (CSSLB) was designed and manufactured for the required tests that permitted the preparation of fully and partially saturated sand specimens (up to 455 mm in height). Through the use of a shaking table, the specimens were subjected to uniform cyclic or earthquake shear strain time histories. The box incorporated pore pressure transducers (PPTs) for the measurement of generated excess pore pressures, linear variable displacement transducers (LVDTs) for the measurement of the induced shear strains, and bender elements and bending disks for the measurement of shear and compression wave velocities of the prepared specimens. The CSSLB described in this paper can be scaled up for strain-controlled testing of larger specimens of sands, as well as gravels, silts, and reconstituted clays.

This paper summarizes the details of the CSSLB and presents the results of numerical analyses to demonstrate the minimal boundary effects of the side walls of the box. The paper also includes typical results of tests performed to illustrate the successful application of the CSSLB under cyclic or transient shear strains. Details of the instrumentation and test results for wave velocity measurements and the liquefaction behavior of fully and partially saturated sands can be found elsewhere (Ortakci 2007; Deniz 2008; Eseller-Bayat 2009).

### Concept Design of the Liquefaction Box

The CSSLB was utilized to conduct tests under cyclic and earthquake-induced shear strains. It was shown by Dobry et al. (1982)

Manuscript received February 23, 2012; accepted for publication March 5, 2013; published online March 22, 2013.

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that excess pore pressure generation is more closely related to cyclic strains induced by an earthquake, rather than cyclic stresses. To achieve this goal, a shaking table was used to induce the controlled shear strains.

In Fig. 1, schematic drawings of the experimental setup and the CSSLB are presented that illustrate the concept of the design of the box and the mechanism through which shear strain was applied to the sand specimens. The CSSLB consisted of two Plexiglas rotating walls, the tops of which were fixed against translation through a metal bar support. The bottoms of the two rotating walls rested on a flexible sealant and hinges, which were used to connect them to the base plate of the box. The commercially available Sikaflex-15LM is a high-performance, low-modulus, elasto-

meric sealant that can permit joint movements of 100 % and 50 % in extension and compression, respectively. The other two walls of the box were rigid vertical Plexiglas walls that were fixed to the base plate. The rotating walls were connected to the rigid walls using the flexible sealant, which allowed relative translations between the rotating and rigid walls. The entire assembly of the box rested on a shaking table. When the shaking table was displaced by an amount  $d$  to the left (as shown in Fig. 1), the base plate and the two rigid walls translated by the same amount. The two rotating walls rotated, and thus the sand specimen experienced an average shear strain of  $\gamma = d/H$  as shown in Fig. 1(c). The use of a shaking table to induce translation of the base plate permits the convenient application of cyclic shear strains of varying amplitudes and frequencies, as well as typical real earthquake (transient) shear strain time histories.

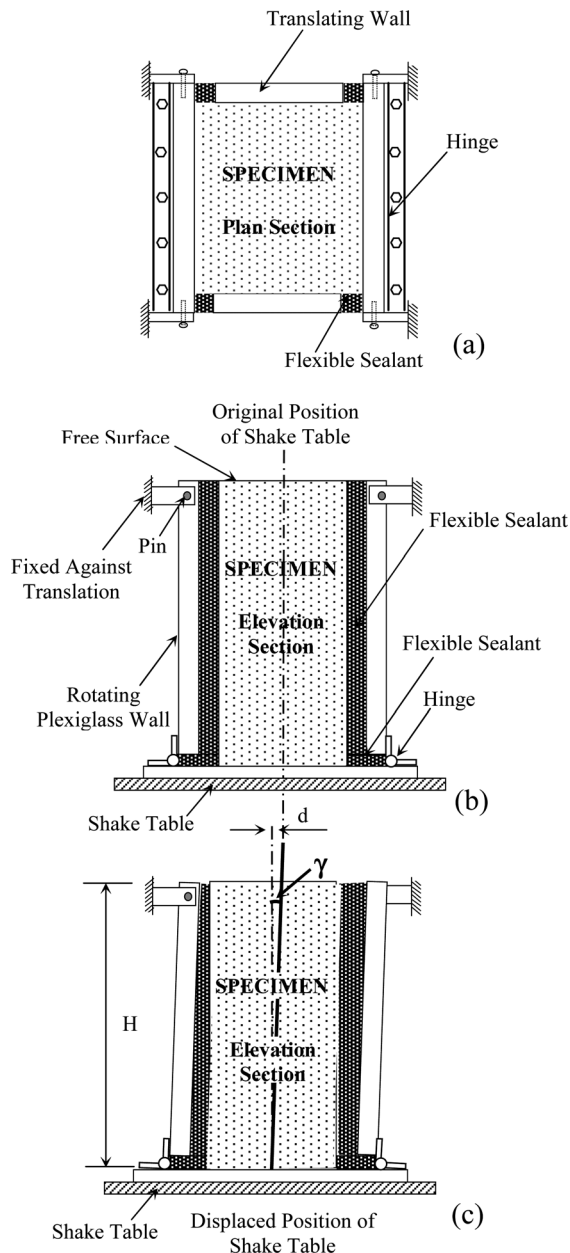


FIG. 1—Simple shear mechanism for the CSSLB: (a) plan section before shearing, (b) elevation section before shearing, and (c) elevation section after shearing by displacing shaking table.

## Details of Cyclic Simple Shear Liquefaction Box

PPTs (Druck PDCR81, 5 psi capacity) were used to measure excess pore pressures induced by the applied shear strain time histories. LVDTs (RDP DCTH400AG, 10 mm stroke) were used to measure the displacement  $d$  in order to calculate the applied shear strain. Bender elements (PSI-5A4E, 31.8 mm ( $l$ )  $\times$  12.7 mm ( $w$ )  $\times$  0.51 mm ( $h$ )) and bending disks (PSI-5A4E, 31.8 mm ( $d$ )  $\times$  0.41 mm ( $h$ )) were used to measure shear and compression wave velocities within the specimen to confirm uniformity of density, as well as to explore the use of wave velocities as a means of measuring the degree of saturation (Deniz 2008; Eseller-Bayat 2009). To evaluate the sidewall boundary effects on the shear strains induced in the specimen, initially two heights (490 mm and 680 mm) were considered for the design of the CSSLB. Numerical model analyses that are described in the next section led to the conclusion that the taller design did not offer any advantages with respect to minimizing boundary effects; instead, a taller box would create logistical problems regarding specimen preparation, instrumentation, and box portability. Therefore, the final design of the CSSLB was based on a 490 mm height as shown in Fig. 2. Figure 2 also shows that the typical dimensions of a sand specimen in the CSSLB were 190 mm  $\times$  300 mm  $\times$  455 mm. Figure 3 shows photographs of the CSSLB empty and with a prepared specimen.

## Numerical Modeling of Sand in Cyclic Simple Shear Liquefaction Box

Numerical modeling of typical sand specimens in the CSSLB in both two-dimensional (2-D) plan and elevation sections was performed using the computer software FLAC (2005). In the models, constant boundary forces ( $F$ ) were applied at the center of the top of the rotating side walls, inducing displacements in the plan sections and shear strains in the elevation sections within the sand specimens.

Figure 4 shows the FLAC mesh for the plan section of the CSSLB that was used to investigate the displacement patterns caused by the application of the forces  $F$  on top of the Plexiglas

rotating walls. The outsides of the two fixed walls were restricted against translation, whereas along the inside of the two fixed walls interface elements were used to model potential slip between the specimen and the walls. Figure 5 shows the FLAC mesh for the

elevation section of the CSSLB that was used to investigate the shear strain patterns induced within a sand specimen. The bottom of the mesh represents the base of the box with fixed boundary nodes. The flexible sealants at the bottoms of the rotating walls

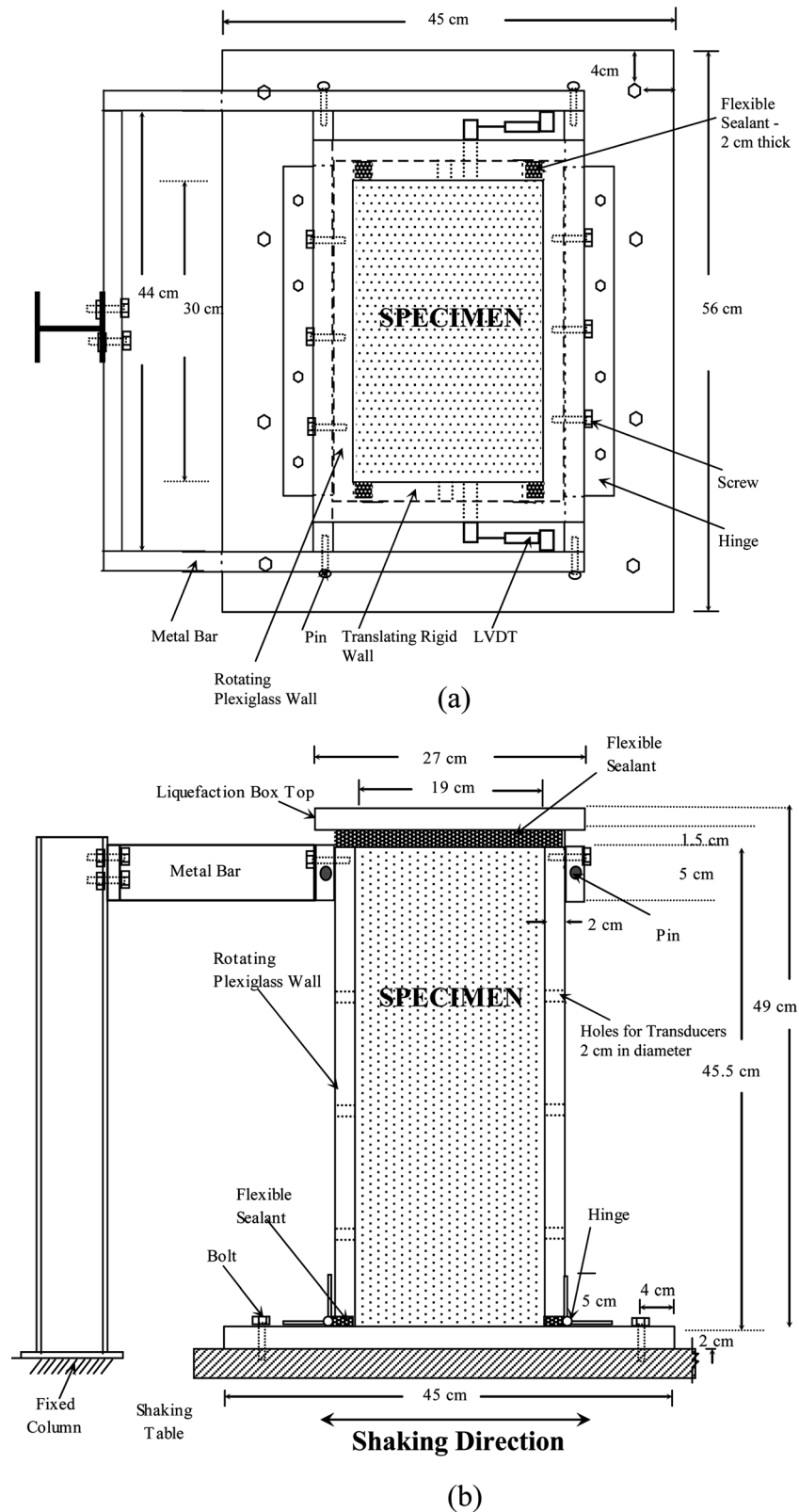


FIG. 2—Details of the CSSLB: (a) plan section; (b) elevation section.

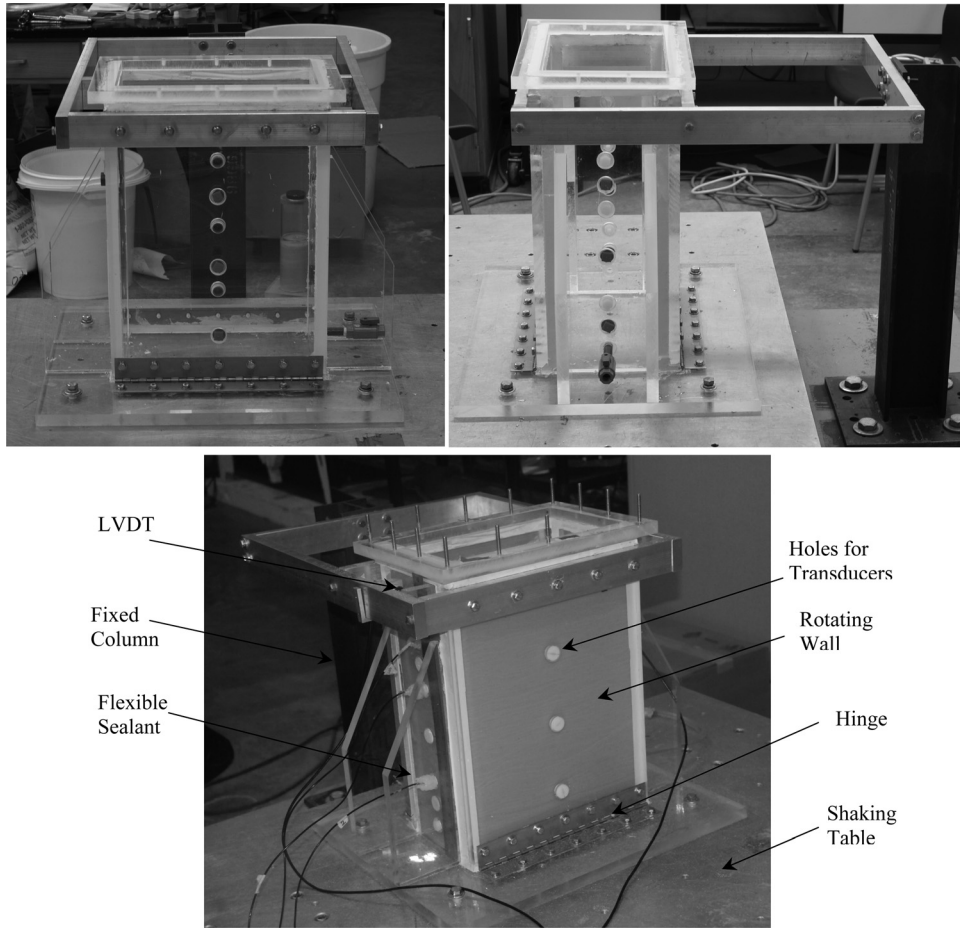


FIG. 3—Photographs of the CSSLB empty and with sand specimen.

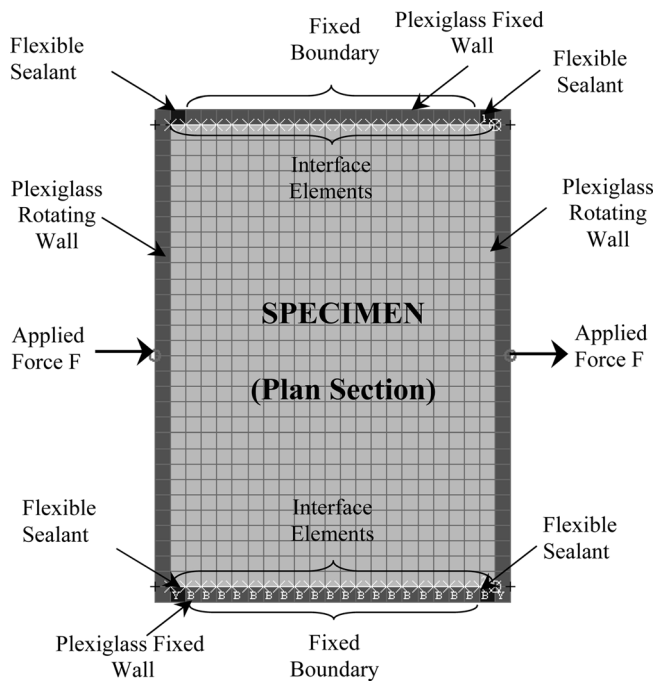


FIG. 4—FLAC mesh for plan section of the CSSLB.

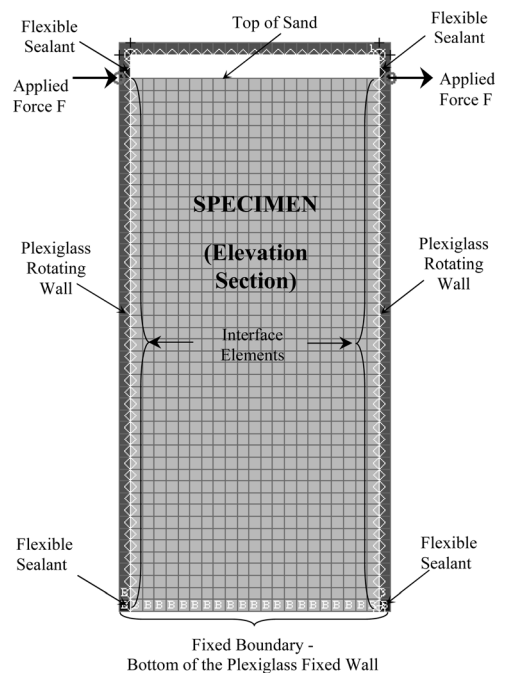


FIG. 5—FLAC mesh for elevation section of the CSSLB.

TABLE 1—Average values for material properties used in 2-D FLAC model.

Material	Model	Shear Modulus $G$ , kPa	Elastic Modulus $E$ , kPa	Poisson's Ratio $\nu$	Normal and Shear Stiffness $K$ , kPa	Cohesion $c$ , kPa
Sand	Elastic	$1.00 \times 10^4$	27 300	0.3	—	—
Plexiglas	Elastic	$1.00 \times 10^8$	$2.60 \times 10^8$	0.3	—	—
Flexible sealant	Elastic	86	250	0.45	—	—
Sand–Plexiglas interface	Interface elements	—	—	—	$3.65 \times 10^8$	0.8

allowed rotation under the applied forces  $F$ . Along the boundaries between the Plexiglas rotating walls and the specimen, interface elements were again used to model slip.

In the FLAC models, elastic elements were used to represent sand, Plexiglas, and flexible sealant, and interface elements were used for the sand–Plexiglas interfaces. Table 1 provides average values of the material properties used for these elements.

The shear modulus of the sand at small strains  $G_{max}$  was estimated using the empirical relationship described by Seed et al. (1986). A shear modulus of  $1 \times 10^4$  kPa was calculated using the effective stress at middle depth, and a value of  $(K_2)_{max} = 40$  was assumed for the loose sand. The shear modulus of the Plexiglas was selected as  $1 \times 10^8$  kPa such that under the static loading conditions, excessive bending of the side walls was prevented in the numerical modeling.

The flexible sealant was modeled using elastic elements that permitted significant contraction and elongation to take place between the Plexiglas walls. The elastic modulus and Poisson's ratio of the flexible sealant were obtained from the specification sheets of the product (Sikaflex-15LM, Sika Corporation, Lyndhurst, NJ). The shear and normal stiffness values of the interface elements were computed using the formulations described in the FLAC user manual and the values of the shear modulus and Pois-

son's ratio of Plexiglas. In addition, an average value of cohesion  $c = 0.8$  kPa was assigned to model slip along interface elements between the Plexiglas walls and sand.

Recognizing the uncertainty present in the estimated material properties, we performed the FLAC analyses for average and ranges of values. In this sensitivity analysis, the following variations in the material properties were considered: for the shear modulus of sand,  $G = 0.5 G$ ,  $G$ , and  $3 G$ ; for interface element cohesion,  $c = 0$ ,  $0.75 c$ ,  $c$ ,  $1.5 c$ , and  $2 c$ ; for the elastic modulus of the flexible sealant,  $E = 0.5 E$ ,  $E$ , and  $2 E$ .  $G = 0.5 G$  can also be considered for the effect of shear strain on the reduction of  $G_{max}$  in the soil specimen. Because interface response is modeled as an elastoplastic behavior, the level of the applied force  $F$  can influence the behavior of the interface elements. For this reason the applied force  $F$  was varied such that  $F = 0.1 F$ ,  $0.5 F$ ,  $F$ , and  $2 F$ . Figure 6 presents computed sand displacements along three lines within the plan section of the CSSLB using average values of the material properties ( $G$ ,  $c$ ,  $E$ , and  $F$ ). The locations of these lines correspond to nodes  $i = 6$ ,  $13$ , and  $20$ . The displacements along these three lines and at the sand–Plexiglas interfaces are almost identical, indicating that the two rigid walls have a negligible effect on the displacement pattern within the sand specimen. Results from the sensitivity analyses indicated that interface

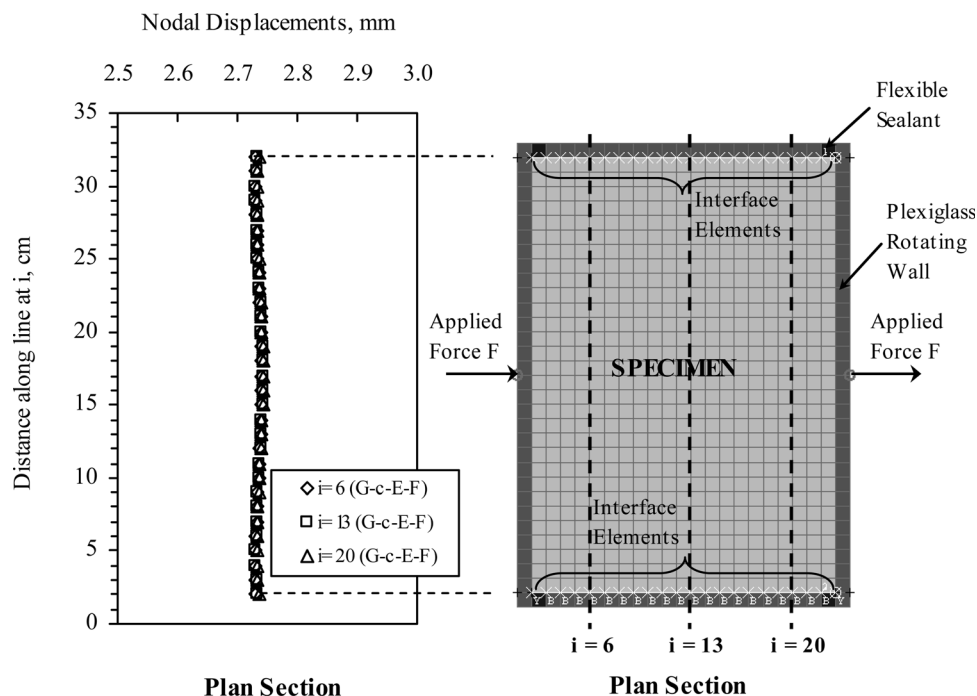


FIG. 6—Displacements along three lines ( $i = 6, 13, 20$ ) within the plan section of the CSSLB for average material properties.

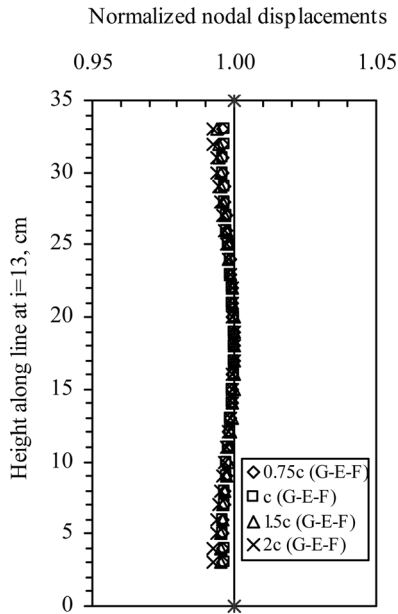


FIG. 7—Normalized displacements along the line at  $i=13$  within the plan section of the CSSLB for variable  $c$  values.

cohesion can influence the behavior of the sand near the Plexiglas walls. Figure 7 shows the displacements normalized with respect to the displacements at the center of the specimen. The effect of the sidewalls slightly increases as the interface cohesion increases,

but it remains negligible. The maximum effect of the sidewalls is that the displacements are reduced by about 1 % near the sidewalls relative to the displacements near the center of the specimen. It is concluded that because of low frictional resistance between the Plexiglas and the sand, the sidewall effects are negligible.

Figure 8 presents results from analysis of the elevation section of the CSSLB using average material properties ( $G$ ,  $c$ ,  $E$ , and  $F$ ). The displacements with depth and along three vertical lines at  $i=6$ , 13, and 20 are almost identical. Along a vertical line, the difference between the calculated displacements of two consecutive nodes was divided by the distance between the nodes to calculate shear strains with depth, as also shown in Fig. 8. It is observed that the shear strains were constant from the top of the sand specimen down to about 5 cm from the bottom of the CSSLB. Figure 9 shows shear strain results from sensitivity analyses of the elevation section in which the interface cohesion was varied. The shear strain results with depth indicate the same pattern as observed in analyses using the average material properties ( $G$ ,  $c$ ,  $E$ , and  $F$ ), shown in Fig. 8.

It is noted that the CSSLB induces, in addition to horizontal shear strains, small vertical strains resulting in slight vertical deformations along the surface of a specimen. For this reason, measurements of pore pressure were made below 5 cm from the surface of a specimen.

In summary, numerical analyses indicate that the CSSLB can induce controlled shear strains in sand specimens. Only near the top and bottom 5 cm of a specimen are boundary effects pronounced, and therefore all transducers are placed between these zones.

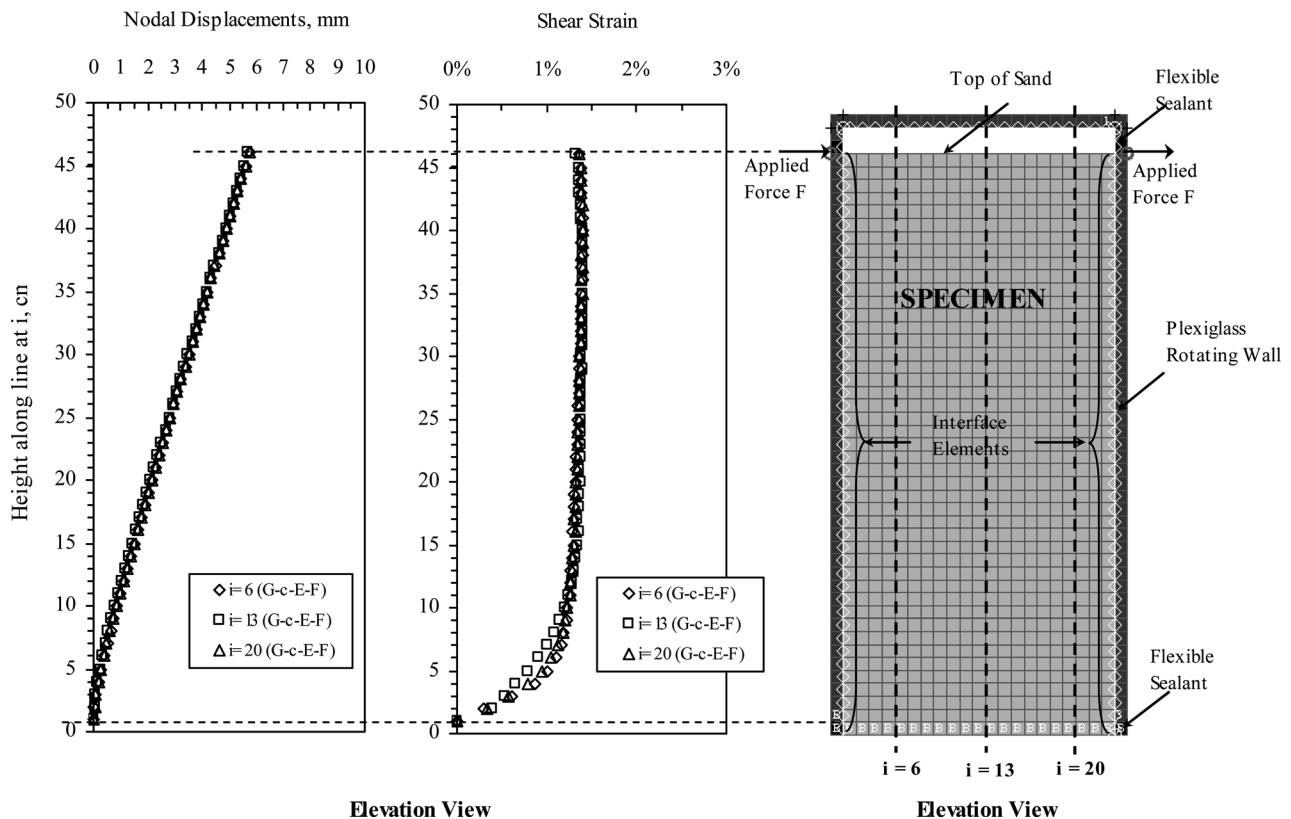


FIG. 8—Displacements and shear strains along three lines ( $i=6$ , 13, 20) within the elevation section of the CSSLB for average material properties.

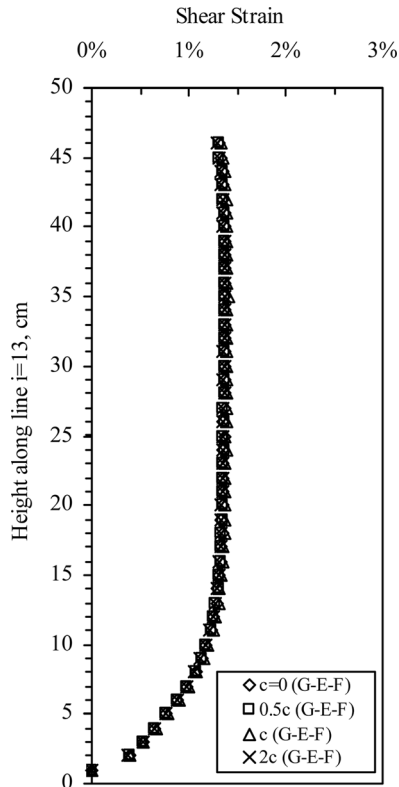


FIG. 9—Shear strains along the line at  $i=13$  within the elevation section of the CSSLB for variable  $c$  values.

### Liquefaction Tests Using Cyclic Simple Shear Liquefaction Box

To demonstrate the application of the CSSLB in laboratory investigations of liquefaction of sands, two typical test results are presented. The first test was on a sand specimen subjected to cyclic shear strains. The second test was on a sand specimen subjected to a typical earthquake-induced shear strain time history.

Figure 10 presents the results from the cyclic shear strain test. A fully saturated sand specimen was prepared via a wet pluviation technique in which dry Ottawa sand was rained into a CSSLB that was partly filled with water. This procedure can result in consistently uniform, low initial relative density ( $D_r=20\%$  to  $30\%$ ) specimens as reported by the authors (Eseller-Bayat et al., accepted and published online on August 8, 2012; scheduled to be published in June 2013). The sand used (ASTM graded C778) was uniform in gradation with a coefficient of uniformity  $C_u$  of 1.1 and a  $D_{10}$  of 0.67 mm. The maximum and minimum void ratios of the sand were 0.80 and 0.50, respectively. PPTs were placed at three different elevations within the sand specimen through holes on the fixed side walls of the CSSLB. A cyclic shear strain with an amplitude of 0.2 % and a frequency of 4 Hz was applied using the shaking table. The frequency of 4 Hz, which is higher than the typical 2 Hz motions used in shaking table tests (Dobry and Abdoun 2011), was selected to minimize the dissipation of excess pore pressures generated during shaking near the top of the specimen. The LVDT record expressed in terms of shear strain is plotted in Fig. 10(a). In Fig. 10(b), the pore pressure records are plotted in terms of centimeters of water. The initial

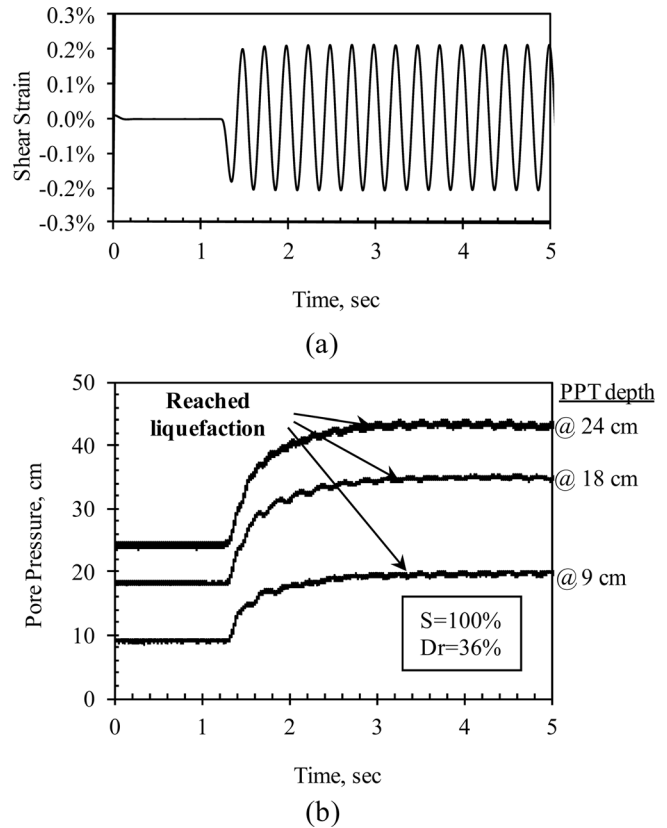


FIG. 10—(a) Cyclic shear strain time history. (b) Pore water pressure responses from three pore pressure transducers (PPTs).

readings are the hydrostatic pressure heads at the PPT locations in the fully saturated sand specimen. As the cyclic shear strains were applied, all three PPTs recorded increasing pore pressures to a maximum value associated with liquefaction of the sand. Liquefaction is defined as the point at which the excess pore pressure is equal to the initial vertical effective stress at the location of a PPT.

Figure 11 shows results from a similar test in which the shaking table was used to induce a typical earthquake shear strain time history. Figure 11(a) shows a time history that was generated using the Treasure Island acceleration record from the 1989 Loma Prieta earthquake and a soil profile of 15 m of sand. Figure 11(b) shows the pore water pressures measured in the sand specimen. The pore water pressure development corresponded very well with the applied shear strains, indicating the ability of the CSSLB to induce the applied strains. This is indirect evidence that when a shear strain time history is externally applied to the CSSLB, it can induce the desired shear strain pattern within a specimen.

These two typical liquefaction test results, and many others conducted as part of this research, demonstrate the efficiency and successful utilization of the CSSLB in testing relatively large sand specimens under strain-controlled conditions in which the shear strains can be either cyclic or transient, as in a real earthquake excitation.

### Limitations of the Cyclic Simple Shear Liquefaction Box

The CSSLB described in this paper offers significant opportunities for testing soil specimens under cyclic and transient shear strain

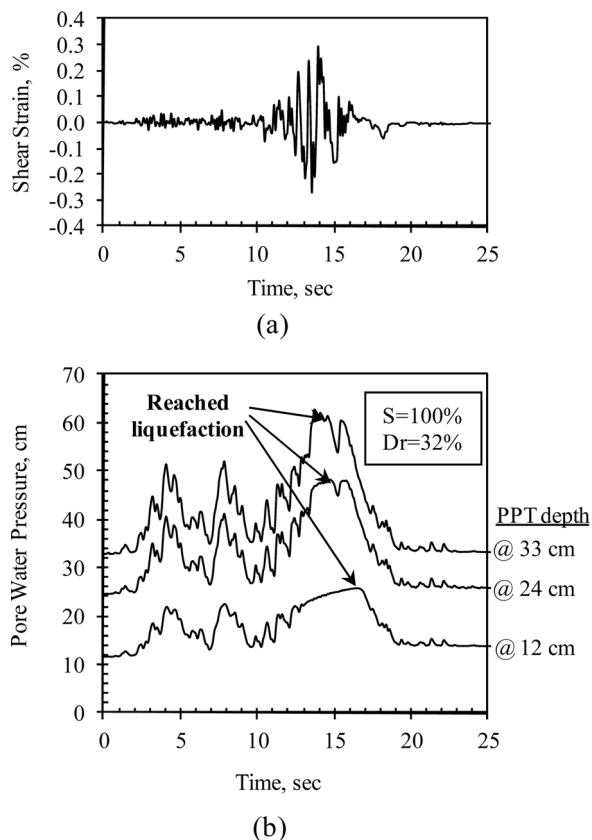


FIG. 11—(a) Earthquake shear strain time history. (b) Pore water pressure responses from three pore pressure transducers (PPTs).

time histories. It is recognized that the box has a number of limitations that should be considered in its applications. The shear strains near the top and bottom of the box (5 cm) can be non-uniform, requiring instruments to be placed between these zones. Because of the free surface of a specimen, drainage can occur, resulting in reduced excess pore pressures, especially for applied cyclic loads with frequencies less than 2 Hz. The box can accommodate the placement of lead weights on top of a specimen to increase effective stresses. The maximum overburden stress achieved in this research was 9.6 kPa (Eseller-Bayat et al., accepted and published online on August 8, 2012; scheduled to be published in June 2013). According to the strength properties of the flexible sealant and its adhesion with Plexiglas, the limiting shear strain that could be applied without damaging the joints was calculated as 2 %.

## Summary and Conclusions

To meet the need for laboratory testing of relatively large soil specimens under controlled shear strain time histories, a special liquefaction box, the CSSLB, was designed and manufactured. The box accommodates pore water pressure transducers and linear variable displacement transducers. The maximum size of a sand specimen that the CSSLB can accommodate is 190 mm × 300 mm × 455 mm. The design of the box and the mechanism for applying shear strain time histories are based on the utilization of two rotating walls connected to two translating rigid walls using a

flexible sealant and a shaking table used to induce shear strains within a specimen.

Numerical analyses of plan and elevation sections of sand specimens in the CSSLB were conducted in order to evaluate potential side wall and base plate effects on the induced shear strains within a specimen. The results indicate that except in the top and bottom 5 cm of the box, the shear strains are uniform.

Typical test results showing pore pressure responses in saturated loose sand specimens have been presented and show that cyclic and earthquake shear strain time histories were induced. In both tests, the pore pressure responses followed closely the pattern of the applied shear strains up to the point of liquefaction. These and other test results have demonstrated that the CSSLB with its instrumentation is well suited for cyclic or transient strain-controlled tests on relatively large soil specimens.

It is noted that the CSSLB can be scaled up for shear strain-controlled testing of much larger specimens, not only of sands, but also of other soils, including gravels, silts, and reconstituted clays.

## Acknowledgments

This research was funded by the National Science Foundation through Geoenvironmental Engineering and Geohazard Mitigation Program under Grant No. CMS-0509894. The support of the NSF and Program Director Dr. Richard J. Fragaszy is greatly appreciated. The contributions to this research by undergraduate civil engineering students Kaitlin Vacca and Olivia Deterling and graduate students R. Oguz Deniz, Fritz Nababan, and Ata Firat Karamanli are acknowledged. Special appreciation is expressed to Dr. David Whelpley and Michael MacNeil for their valuable support in the design and manufacturing of the liquefaction box.

## References

- Bjerrum, L. and Landva, A., 1966, "Direct Simple Shear Tests on a Norwegian Quick Clay," *Geotechnique*, Vol. 16, No. 1, pp. 1–20.
- Budhu, M., 1985, "Lateral Stresses Observed in Two Simple Shear Apparatus" *J. Geotech. Eng.*, Vol. 111, No. 6, pp. 698–711.
- Dashti, S., Bray, J. D., Pestana, J. M., Riemer, M., and Wilson, D., 2010, "Centrifuge Testing to Evaluate and Mitigate Liquefaction-Induced Building Settlement Mechanisms" *J. Geotech. Geoenviron. Eng.*, Vol. 136, p. 918.
- Deniz, O., 2008, "Bender Elements and Bender Disks for Measurement of Shear and Compressional Wave Velocities in Large Sand Specimens," M.S. thesis, Northeastern University, Boston, MA.
- Dobry, R. and Abdoun, T., 2001, "Recent Studies on Seismic Centrifuge Modeling of Liquefaction and Its Effects on Deep Foundations," *Proceedings of the Fourth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, San Diego, CA, March 26–31, University of Missouri-Rolla, Rolla MO.
- Dobry, R. and Abdoun, T., 2011, "An Investigation into Why Liquefaction Charts Work: A Necessary Step toward Integrating the States of Art and Practice," *Proceedings of the 5th*



- International Conference on Earthquake Geotechnical Engineering*, Santiago Chile, January 10–13 2011, Chilean Geotechnical Society, pp. 13–45.
- Dobry, R., Ladd, R. S., Yokel, F. Y., Chung, R. M., and Powell, D., 1982, *Prediction of Pore Water Pressure Buildup and Liquefaction of Sands during Earthquakes by the Cyclic Strain Method*, U.S. Department of Commerce, Washington, D.C.
- Eseller-Bayat, E. E., 2009, “Seismic Response and Prevention of Liquefaction Failure of Sands Partially Saturated Through Introduction of Gas Bubbles,” Ph.D. dissertation, Northeastern University, Boston, MA.
- Eseller-Bayat, E., Gokyer, S., Yegian, M. K., and Alshawabkeh, A., “Liquefaction Response of Partially Saturated Sands: Experimental Results,” *J. Geotech. Geoenviron. Eng.* (accepted and published online on August 8, 2012; scheduled to be published in June 2013).
- FLAC Version 5.0 User's Guide*. (2005). Itasca Consulting Group, Inc., Minneapolis, MN.
- Moussa, A. A., 1974, “Shearing Resistance of Sand to Repeated Sea Storms,” *Internal Report No. 51505-2*, Norwegian Geotechnical Institute, Oslo, Norway.
- Ortakci, A. E., 2007, “Design and Manufacturing of a Cyclic Simple Shear Liquefaction Box (CSSLB),” M.S. thesis, Northeastern University, Boston, MA.
- Roscoe, K. H., 1953, “An Apparatus for the Application of Simple Shear to Soil Samples,” *Proceedings of the Third International Conference on Soil Mechanics and Foundation Engineering*, Vol. 1, ICOSOMEF, London, pp. 186–191.
- Seed, H. B., Wong, R. T., Idriss, I. M., and Tokimatsu, K., 1986, “Moduli and Damping for Dynamic Analyses of Cohesionless Soils,” *J. Geotech. Eng.*, Vol. 112, No. 11, pp. 1016–1032.
- Thevanayagam, S., Kanagalingam, T., Reinhorn, A., Tharmendhira, R., Dobry, R., Pitman, M., Abdoun, T., Elgamal, A., Zeghal, M., Ecemis, N., and El Shamy, U., 2009, “Laminar Box System for 1-g Physical Modeling of Liquefaction and Lateral Spreading,” *Geotech. Test. J.*, Vol. 32, No. 5, pp. 438–449.
- Yegian, M. K., Eseller-Bayat, E., Alshawabkeh, A., and Ali, S., 2007, “Induced Partial Saturation for Liquefaction Mitigation: Experimental Investigation,” *J. Geotech. Geoenviron.*, Vol. 133, No. 4, pp. 372–380.