

THIS IS A PREPRINT --- SUBJECT TO CORRECTION

Lateral Soil Resistance - Displacement Relationships for Pile Foundations in Soft Clays

By

Missak Yegian and Stephen G. Wright, The U. of Texas at Austin

© Copyright 1973

American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc.

Offshore Technology Conference on behalf of the American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc., American Association of Petroleum Geologists, American Institute of Chemical Engineers, American Society of Civil Engineers, American Society of Mechanical Engineers, Institute of Electrical and Electronics Engineers, Inc., Marine Technology Society, Society of Exploration Geophysicists, and Society of Naval Architects & Marine Engineers.

This paper was prepared for presentation at the Fifth Annual Offshore Technology Conference held in Houston, Tex., April 29-May 2, 1973. Permission to copy is restricted to an abstract of not more than 300 words. Illustrations may not be copied. Such use of an abstract should contain conspicuous acknowledgment of where and by whom the paper is presented.

ABSTRACT

This paper describes a new procedure, based on the finite element method, for developing the lateral soil resistance-displacement relationships (p-y curves) for single or group pile foundation under short term static loading. In this procedure a nonlinear stress-strain relationship is employed for the soil surrounding the pile and a special cylindrical element is developed to model the soil-pile interface characteristics. This interface element is also characterized by a nonlinear relationship to represent the soil movements around and immediately adjacent to the pile. The procedure described is capable of accommodating complex pile geometries and loading sequences, as well as a variety of soil types and soil-pile interface properties.

Typical results obtained using the finite element model are presented, and where appropriate, these are compared with the results of conventional procedures for developing p-y curves and predicting lateral soil resistance. The finite element model has been found to provide a powerful tool for evaluating soil-pile interaction under lateral loading. It

References and illustrations at end of paper.

provides the capability to investigate more complex problems than previously considered by existing procedures and to evaluate these in more detail and on a more fundamental level than before. It has been found that the soil-pile interface properties may have a significant influence on the lateral soil-resistance displacement relationship, and thus, an improved definition of this interface may be appropriate in future analyses.

INTRODUCTION

Many pile supported offshore structures are subjected to relatively significant lateral loads, and consequently their pile foundations must be designed to withstand loading of this type. Most of the conventional analysis procedures used to evaluate the behavior of laterally loaded piles require, among other information, that the soil resistance-displacement relationship, describing the soil-pile interaction, be represented by some simple numerical means. Typically the soil resistance-displacement relationships are referred to as "p-y curves" and are developed employing empirical procedures based on the soil properties and pile geometry. Considerable experience has been obtained with these procedures and in most instances the results appear to have been successful. However, the

present procedures possess limitations which may be significant for some types of pile foundations and loading conditions. Because the present procedures for developing soil-pile interaction curves are empirically derived and have been based largely on a single pile, considerable uncertainty exists when attempts are made to apply these procedures to pile groups consisting of several piles in varying arrangements. While a limited number of experimental studies have been conducted to evaluate the lateral soil resistance displacement response of pile groups, the results are generally restricted to the specific pile group arrangement used in the study.

The purpose of this paper is to present an alternate analytical procedure for developing the soil resistance-displacement relationships for laterally loaded piles. This procedure is based on a two dimensional model employing the finite element method with nonlinear material properties for analysis. For the present paper only static, short-term loading conditions in soft saturated clays are considered.

p-y RELATIONSHIPS

When a pile is subjected to a lateral load, as illustrated in Fig 1, it will displace laterally until equilibrium is established with respect to the stresses and displacements of both the soil and the pile at each point along the pile. Initially, prior to any lateral displacement, the pile is at each level subjected to a uniform state of stress around its perimeter as illustrated by the typical cross-section shown in Fig 2a. As the pile is then displaced laterally through a distance, y , the state-of-stress around the pile changes to a non-uniform distribution similar to the one shown in Fig 2b. This non-uniform pressure distribution is commonly characterized by a resultant lateral force (per unit length of pile), p , and the relationship describing the complete soil-pile resistance as a function of the lateral displacement of the pile is referred to as a "p-y curve". A number of these curves are necessary to define the soil-pile resistance at all levels along the pile.

In developing p-y relationships and employing such relationships in the analysis of laterally loaded piles, it is generally assumed that the soil resistance-displacement relationship (p-y curve) at any level along the pile is independent of the pile displacement and soil response at points above and below the specific point of interest. This assumption, which essentially uncouples the influence of soil displacements along the length of the pile, considerably simplifies the analysis of piles subjected to lateral loading.

Because of the nonlinear stress-strain behavior of most natural soil deposits the corresponding p-y curves are nonlinear in form as illustrated in Fig 3. McClelland and Focht (1958) have shown that these curves are similar in form to the stress-strain curves obtained from laboratory compression tests and it is expected that a close degree of correspondence must exist between the two. However, a precise relationship between the two is difficult to establish from a theoretical viewpoint because of the pronounced nonlinear behavior. Most previous attempts have been based on the combined results from the theories of elasticity and plasticity with necessary empirical adjustments being made to obtain nonlinear p-y relationships from the stress-strain and strength data for the soils involved. This approach appears to have worked reasonably well in practice and until recently the absence of more rigorous theoretical procedures for analysis has made this the only apparently rational approach for developing p-y relationships for laterally loaded piles.

Although previous efforts in developing p-y curves have been limited by available theoretical procedures, the significant development of the finite element method during the past several years has provided a powerful and relatively new tool for analysis. The finite element method appears well-suited to the analysis of problems involving nonlinear material properties, including the class of problems represented by laterally loaded piles, and specifically, their corresponding p-y relationships. In addition, the flexibility provided by the finite element method for considering complex geometries and loading sequences offers almost unrestricted capabilities for evaluating pile group arrangements.

ANALYTICAL MODEL

Soil-Pile Model

For purposes of analysis the pile and surrounding soil may be represented by a plane cross-section taken horizontally through the pile as illustrated for a single pile in Fig 4. At a sufficient distance, r , away from the pile the magnitude of the soil displacements resulting from lateral movement will be relatively small and for practical purposes may be neglected. Thus, a fixed cylindrical boundary of radius, r , may be established as shown by the broken line in Fig 4. The zone of pile influence, as represented by the radius, r , is related directly to the diameter of the pile (D). From the results of analyses of the type described herein, in which the fixed boundary r was varied, the radius of the zone of the pile influence has been found to be approximately 8 times the pile diameter. For pile

groups the zone of influence is somewhat more complex and is related to the pile sizes and group configuration. Typical fixed boundary limits for several pile groupings are shown later in this paper.

In most planar analyses of single piles and pile groups at least one symmetry boundary exists. This boundary may be used to further simplify the analytical model and significantly reduce the computational effort required. For example, considering a single pile, the pattern of stresses and displacements are symmetrical about a plane passing through the longitudinal axis of the pile and oriented in the plane of lateral pile displacement. The line of symmetry formed by this plane is illustrated in Fig 5. Along this boundary there are no shear stresses and the soil displacements perpendicular to the direction of pile displacements are zero. This condition is represented by the "rollers" shown for the single pile model in Fig 5. For grouped piles the lines of symmetry are located somewhat differently and will depend on the geometry of the pile group and the direction of pile motion. Typical symmetry boundaries for grouped piles are illustrated in later sections of this paper.

In addition to the symmetry boundaries, several antisymmetry boundaries exist provided that certain stress and strain characteristics are applicable for the soil surrounding the pile. That is, the modulus and strain in the soil must depend only on the changes in shearing stresses, and the stresses in the soil must not approach levels where a tensile failure occurs and the soil separates from the pile. The analyses presented in this paper were performed for depths believed to be below the depth where tensile failure might occur and thus do not preclude the existence of antisymmetry boundaries. Further, because the stress-strain model employed in the analyses was appropriately assumed to depend only on the magnitude of the shearing stresses, the conditions required for the existence of an antisymmetry boundary were met. While use of this boundary is not necessarily a prerequisite for the use of the finite element model described herein, consideration of this boundary considerably reduced the computational effort required for the analyses.

For a single pile the plane of antisymmetry lies perpendicular to the symmetry plane and passes through the longitudinal axis of the pile. This plane forms the antisymmetry boundary b-b' shown on the planar cross-section in Fig 6. Along this boundary the lateral displacements of the soil perpendicular to the direction of pile movement are zero and no change in stress occurs normal to the plane represented by the line b-b'. These two conditions of anti-

symmetry are represented schematically in Fig 6 by the "rollers" along the line b-b'.

In performing analyses using a two-dimensional model like the one illustrated in Fig 6, it is necessary to introduce an assumption regarding the stresses or displacements in a direction perpendicular to the plane of interest. For the present analyses plane stress behavior of the cross-section was assumed, an assumption which is equivalent to assuming that no change in vertical stress occurs in the soil surrounding the pile. This assumption appears to be reasonable, particularly at shallow depths encompassing the region where most of the lateral soil resistance is mobilized by laterally loaded piles. In this region the vertical stress is initially equal to the soil overburden pressure and should remain essentially equal to this pressure in the absence of extremely high axial pile loads.

Finite Element Model

Using the above model, computation of the stresses and displacements in the soil surrounding the pile and at the soil-pile interface were performed employing two-dimensional, plane stress finite element analyses. The soil surrounding the pile was represented by two-dimensional quadrilateral elements of the form described by Doherty, Wilson and Taylor (1969) and referred to as the "QM5" element. This element has been found to provide good response characteristics for modeling two-dimensional plane problems. However, at the interface between the soil and the pile two-dimensional quadrilateral elements of this type are inadequate for characterizing the soil-pile interaction and fail to model properly the slippage and flow of soil immediately adjacent to the pile.

In order to model more properly the interface behavior between the soil and the pile it is appropriate to introduce a specialized "slip" element at the interface. Straight line elements of this type have been previously developed by Goodman et al (1968) for modeling the joint interface behavior in jointed rock, and Clough and Duncan (1969) have employed these elements with excellent success for modeling the interaction between soil and concrete retaining structures. For the present problem a similar, but curved, element was developed to represent the cylindrical boundary between the soil and the pile. This element, illustrated schematically in Fig 7a, is of infinitesimal thickness in the radial direction but possesses a finite length encompassing a segment of the pile perimeter. Deformations of the element are restricted in the radial (normal) direction in accordance with the assumption that the soil remains in

contact with the pile; but, tangential (shear) deformation is permitted to occur around the circumference of the pile. The tangential displacement of the interface element, illustrated by the displacement pattern shown in Fig 7b, represents the movement (slip) of the soil relative to the pile and is governed by the interface properties assigned to the element. The normal and shear stresses in the element correspond to the normal and shear stresses transferred to the pile by the soil, and once computed, these stresses may be resolved in the direction of pile displacement (y) to obtain the net resultant force (p).

The computation of stresses at the soil-pile interface was performed using an incremental analysis procedure employing nonlinear material properties. In these analyses the pile was assumed rigid, and for each increment in the analyses a uniform displacement (y) was applied at the perimeter of the pile in the direction of pile movement. The shear and normal stresses at the interface were computed in the finite element analyses and resolved into the net lateral resultant, p , thus establishing for each increment of displacement, y , a single point on a p - y curve for the pile.

SOIL PROPERTIES

The soil properties which influence the p - y curves for a laterally loaded pile and were required for the finite element analyses include the stress-strain relationship for the soil surrounding the pile and the shear stress-soil displacement relationship characterizing the interface behavior at the soil-pile boundary. Nonlinear expressions were used to describe both of these relationships and for each increment of pile displacement an iterative, secant modulus procedure was employed to obtain proper convergence to the correct values of stress and strain. Special procedures were developed and utilized for these iterative analyses and in all instances extremely rapid convergence to the correct values of stress and strain was achieved.

Stress-Strain Properties

The stress-strain properties of saturated clays are for many design purposes appropriately represented by undrained loading conditions. For the present analyses these loading conditions were assumed, and the relationship between shear stress, τ , and shear strain, γ , up to the point of failure ($\gamma = \gamma_f$) was expressed by a hyperbolic relationship of the form illustrated in Fig 8a and written as,

$$\tau = G_i \frac{\gamma}{\left[1 + \gamma \left(\frac{G_i}{s} - \frac{1}{\gamma_f} \right) \right]} \quad (1)$$

In this expression G_i is the initial shear modulus, represented by the initial slope of the stress-strain curve shown in Fig 8a, γ_f is the shear strain at failure, also illustrated in this figure, and s is the undrained shear strength of the soil as typically measured by field vane shear tests or unconsolidated-undrained laboratory triaxial compression tests. In all analyses performed a value of 0.495 was assumed for Poisson's ratio, consistent with the nearly incompressible behavior of saturated clays during undrained loading. Under these conditions the initial shear modulus, G_i , and the shear strain at failure, γ_f , are related to the initial Young's modulus, E_i , and axial strain at failure, ϵ_f , as measured in the conventional triaxial compression test and illustrated by the stress-strain relationship shown in Fig 8b, by the following expressions:

$$G_i \cong E_i/3 \quad (2)$$

$$\gamma_f \cong \epsilon_f \cdot 3/2 \quad (3)$$

The hyperbolic stress-strain relationship given by Eq 1 was utilized to describe the stress-strain behavior of the soil up to the point at which the shear strain, γ_f , was developed and the shear strength of the soil was fully mobilized. Beyond this point the soil was assumed to behave plastically as illustrated by the horizontal line on the stress-strain relationships shown in Figs 8a and 8b.

The nonlinear, hyperbolic expression used to describe the relationship between stress and strain in clay soils was suggested by Konder (1963) and has since been found to be both a convenient and useful tool for use in nonlinear finite element analyses (Duncan and Chang, 1970). The stress-strain relationship for saturated clays in the form described herein was previously employed in an identical manner by Wright and Dunham (1972) in the analyses of sea-floor movements resulting from wave action. Typical ranges in the values of the initial moduli (E_i and G_i) were discussed in relation to offshore soil deposits and where accurate and reliable stress-strain data are not available, correlations between shear strength and modulus values were suggested. Except where noted in the present analyses the initial modulus value, E_i , was related to the shear strength by the relationship,

$$E_i = 100 \cdot s \quad (4)$$

or in terms of the initial shear modulus,

$$G_i = 33.3 \cdot s \quad (5)$$

The corresponding strain at failure, γ_f , was assumed equal to 15 percent.

Interface Properties

Soil properties in addition to those described above are necessary in order to describe the characteristics of the soil-pile interface, as represented by the curved interface elements in the finite element analyses. These consist of an appropriate nonlinear expression describing the relationship between tangential shear stress and relative movement, or "slip," of the soil immediately adjacent to the pile. Following the procedure developed by Clough and Duncan (1969) for modeling the interface properties between soil and concrete in finite element analyses, the shear stress-displacement relationship was expressed by a hyperbolic equation of the form,

$$\tau = K_i \left[\frac{\delta}{1 + \delta \left(\frac{K_i}{\tau_f} - \frac{1}{\delta_f} \right)} \right] \quad (6)$$

In this expression, relating the shear stress, τ , to the relative movement, or "slip," (δ) at the soil-pile interface, K_i is the initial slope of the $\tau - \delta$ relationship, illustrated in Fig 9, and τ_f and δ_f correspond, respectively, to the maximum interface shear resistance and the displacement at which this resistance is first developed. Once the maximum resistance is developed the shear stresses are assumed constant with increased relative soil-pile movement, as represented by the horizontal line on the $\tau - \delta$ relationship shown in Fig 9.

The maximum interface shear resistance (τ_f), or soil-pile adhesion, for steel piles in soft saturated clays is frequently and conveniently related to the undrained shear strength of the surrounding soil. For the present analyses the maximum soil-pile adhesion was assumed directly related to the soil shear strength by an expression of the form,

$$\tau_f = f_c \cdot s \quad (7)$$

where f_c represents a soil strength reduction or adhesion factor, depending in value on the type and condition of the pile material, and s is the undrained shear strength of the soil.

SINGLE PILE ANALYSES

Influence of Interface Properties

In order to establish the influence of the interface properties on the resulting p-y relationships for laterally loaded piles a series of analyses were performed in which

the adhesion factor, f_c , and corresponding relative soil-pile displacement, δ_f , were varied over a range in values. For purposes of analysis a 30-inch diameter pile was assumed and the shear strength of the surrounding soil was selected to be 100 psf. Potyondy (1961) has reported typical values of the adhesion factor ranging from 0.25 for smooth steel piles to 0.50 for rough piles, and for comparative purposes values of 0.0, 0.3, 0.6 and 1.0 were selected. In addition an analysis was performed in which the pile was assumed infinitely rough, corresponding to an infinite f_c value with no slippage of soil immediately adjacent to the pile. The relative displacement, δ_f , at which the maximum adhesion was first developed was assumed to range from 0.02 to 0.10 inch. This range in values appears consistent with the results reported by Chuang and Reese (1968) and is supported by data provided to the authors through the courtesy of Shell Oil Company. For the present analyses the initial slope, K_i , defining the $\tau - \delta$ relationship at the soil-pile interface was defined in terms of τ_f ($= f_c \cdot s$) and δ_f by the relationship,

$$K_i = \frac{\tau_f}{\delta_f} \quad (8)$$

This expression for K_i reduces the hyperbolic relationship for $\tau - \delta$ to a simple bi-linear expression; however, for the present analyses this equation is believed adequate.

For illustrative purposes it is useful to compare the values of the ultimate soil resistances, p_u , corresponding to the p-y relationships computed for the range in interface properties described above. These values are summarized in Table 1. It may be noted that the maximum ultimate resistance corresponds to an infinitely rough pile (infinite f_c) where slippage of the soil immediately adjacent to the pile is restricted, while the minimum load corresponds to a perfectly smooth ($f_c = 0$) pile. The maximum ultimate value is approximately 37 percent higher than the minimum with intermediate values of ultimate resistance corresponding to values of f_c equal to 0.3, 0.6 and 1.0. The p-y relationships corresponding to the range in adhesion factors selected and a value of 0.10 inch for δ_f are illustrated in Fig 10, showing the increased lateral resistance accompanying the increased soil-pile adhesion.

While the ultimate lateral resistance appears to be significantly influenced by the adhesion factor, the relative displacement, δ_f , at which the full adhesion is developed appears from the present analyses to have negligible influence in the range assumed. In the analyses summarized in Table 1 it was

observed that the peak interface shear was developed at relatively small pile displacements and thus was in all cases fully developed when the ultimate resistance was reached, regardless of the value of δ_f . Further, for the analyses performed the values of δ_f had only negligible influence on the shapes of the p-y relationships. On the basis of these analyses and for the conditions investigated (single, large diameter piles) the value of δ_f appears insignificant, provided that the value is selected within the range suggested above and used in these analyses. However, for piles of considerably smaller diameter and grouped piles, the values of δ_f may assume increased significance.

Comparative Analyses

The analyses described above appear to yield reasonable p-y relationships and indicate significant influence of the interface properties. However, no information is provided concerning the accuracy of the finite element procedure as compared to established procedures for predicting p-y curves and to the available experimental data supporting these procedures. As a result a series of analyses were performed to establish the degree of correspondence between the finite element procedure and an already established procedure. For this purpose the criteria proposed by Matlock (1970) was selected. These are based on a considerable amount of experience and experimental data and are believed to be among the most reliable and established procedures available for developing p-y relationships for piles in soft clay. While these are limited to single piles they are judged well-suited to the present comparisons.

For comparative purposes a typical offshore soil profile was selected and actual soil properties from this site were employed in the analyses. In the procedure proposed by Matlock a distinction is made between relatively shallow and great depths and appropriate correction factors are applied to the ultimate lateral resistance depending on these relative depths. In this procedure the ultimate resistance, p_u , is expressed as,

$$p_u = N_p \cdot s \cdot D$$

where s is the soil strength, D is the pile diameter and N_p is referred to as an ultimate resistance coefficient. The value of N_p is adjusted depending on the relative depth below the surface (mudline) and increases from a value of 3 at the surface to 9 at an empirically defined depth, x_r , below which the value of N_p remains constant. Consequently in comparing the results of the finite element analyses with Matlock's criteria it

was judged important to select depths in both "shallow" ($N_p < 9$) and "deep" ($N_p = 9$) regions of the soil profile. For this reasons depths corresponding to 4 and 16 pile diameters were selected for the 24-inch diameter pile analyzed. The respective values of shear strength for the soil profile considered were 160 and 280 psf resulting in calculated N_p values of 6.5 and 9.0.

A limited amount of stress-strain information was available for the soil deposit selected for analysis and was used to establish the values of initial moduli (G_i) and shear strain at failure (γ_f) at the two depths considered. These values, summarized in Table 2, were used to define hyperbolic stress-strain relationships used in the finite element analyses and assumed as the true stress-strain relationship in employing Matlock's criteria. In the finite element analyses the adhesion factor (f_c) and corresponding displacement (δ_f) for the soil-pile interface were assumed equal to 0.3 and 0.10 inch, respectively.

"Shallow" Pile Comparison. The p-y curves obtained for a relative depth of 4 pile diameters (= 8 feet) are compared in Fig 11. Although some deviation between these two curves may be noted at intermediate pile displacements in the range shown, relatively close agreement is found between the ultimate values obtained using Matlock's criteria and the finite element procedure.

At relative depths shallower than the one considered, it is significant to note that the ultimate pile resistance predicted by Matlock's criteria will be increasingly lower than the value obtained from the finite element procedure. This characteristic is believed to be a result, in part, of the soil separating from the pile, a condition not reflected in the finite element analyses described herein. Such behavior may be conveniently modeled in the finite element procedure by adjustments in the interface (slip) element properties allowing unrestricted deformation at the interface in a direction radially away from the pile. However, analyses of these conditions are beyond the intended scope of the current paper.

"Deep" Pile Comparison. The p-y relationships obtained for the pile at a depth of 16 diameters (= 32 feet) are shown in Fig 12. In this instance reasonable agreement is developed by the pile, significant disagreement may be noted in the soil resistance values (p) computed by the two procedures. The ultimate resistance computed by Matlock's criteria (410 lbs/in) is approximately 29 percent greater than the corresponding value (318 lbs/in) computed by the finite element procedure. Although closer agreement may be obtained between the two procedures by employing values

of the adhesion factor which are greater than the value used ($f_c = 0.3$) in the finite element analyses, infinite soil-pile adhesion must be assumed to achieve the closest agreement. In this extreme case the ultimate resistance computed from the finite element analyses is approximately 400 lbs/in.

Similar variations in the ultimate resistances computed by the finite element procedure and Matlock's criteria were also found in the analyses of the 30-inch diameter pile previously described and illustrated in Fig 10. The p-y relationship from this figure corresponding to an adhesion factor of 0.3 is compared with Matlock's relationship in Fig 13. The ultimate resistance in this instance is approximately 33 percent greater by Matlock's criteria and close similarity between the comparative analysis for the 24-inch and 30-inch diameter piles illustrated in Figs 12 and 13 may be noted. The soil strength profiles and stress-strain properties differed for these two sets of analyses; however, both correspond to "deep" pile conditions (i.e. $N_p = 9$).

The apparent discrepancies between the ultimate pile resistances computed for "deep" pile conditions by the two procedures employed do not appear completely reconcilable in terms of adjustments in the interface properties and increasing the adhesion factor used in the finite element analyses, although somewhat closer agreement may be obtained by such adjustments. As previously discussed, the finite element analyses were performed assuming two-dimensional plane stress behavior with no change in stress in the vertical direction (normal to the plane of reference). This assumption, while reasonable at shallower depths, may be somewhat unrealistic at greater depths. At these greater depths increased confinement of the soil may lead to restricted deformation in the vertical direction and as an extreme case a plane strain condition of no vertical strain might be approached. In this latter case considerably higher soil resistance would be expected.

To examine the influence of vertical restraint an additional finite element analysis with the 30-inch diameter pile and soil conditions was performed assuming plane strain behavior of the soil-pile cross section. The corresponding p-y curve is compared in Fig 14 with those obtained by Matlock's criteria and the plane stress finite element analyses. It is interesting to note that the criteria proposed by Matlock yield an ultimate pile resistance which is approximately midway between the resistances corresponding to plane stress and plane strain behavior in the finite element analyses. This suggests that neither of the idealized two-dimensional

models (plane stress or plane strain) precisely represents the actual soil behavior.

Summary

On the basis of the above analyses for a single pile it appears that reasonably accurate p-y relationships may be developed with the finite element model, provided that reliable shear strength and soil-pile interface properties are employed. Some disagreement was found between the results obtained using the finite element procedure and those found using Matlock's criteria. However, the significance of these differences appears small, particularly inasmuch as the accuracy of the procedure (Matlock's) used as a basis for comparison, or of any procedure, is impossible to establish. In the analyses performed a variation in the range of interface properties was shown to produce a 37 percent variation in the ultimate pile resistance while differences between the finite element and Matlock's procedures were found in all cases to be less than this value, the maximum difference being 33 percent.

A consistent variation and difference in the shape of the p-y curves has been shown for the two procedures compared. This results from assumptions employed in each. The criteria suggested by Matlock assumes a parabolic shape for the p-y curve, fitting the curve through two empirically derived points corresponding to the ultimate pile resistance and 50 percent of this ultimate resistance. These two points coincide almost precisely with the two points on the corresponding p-y relationship obtained from finite element analyses, provided that infinitely rough pile conditions are assumed and no slippage of the soil is permitted immediately adjacent to the pile. While all p-y curves developed from Matlock's criteria are parabolic in shape, the corresponding relationships developed from the finite element analyses assume an essentially hyperbolic shape as a result of the hyperbolic stress-strain relationship employed. These differences appear insignificant from a practical point of view and either shape for the p-y relationship appears reasonable.

Both procedures considered for developing p-y curves for single piles should result in reasonable values for design of many conventional offshore structures which are subjected to lateral loads. However, the finite element analyses indicate that some caution may be necessary in selecting design values for structures subjected to mudslides and sea-floor instability. Bea (1971) has described the extensive damage to two offshore structures, apparently as a result of a sea-floor slide, and Wright and Dunham (1972) have shown that large lateral soil movements may develop to considerable depths in the ocean floor in areas

where instability is likely to develop. These movements may exert significant lateral forces on deep pile foundations with limiting values of load corresponding to the ultimate soil-pile resistance, p_u . Consequently in slide areas, an underestimate in the value of the ultimate resistance may lead to an underestimate in the lateral forces developed on the pile. It is significant to note that the results shown in Fig 14 yield a value for the ultimate resistance for plane strain conditions (245 lbs/in) which is 31 and 74 percent greater than the corresponding values obtained from Matlock's criteria and the plane stress finite element analyses, respectively. If an infinitely rough pile is assumed these differences are increased respectively to 69 and 78 percent. Errors or overestimates of this magnitude, may result in appreciable errors in the design of pile foundations to resist sea-floor slides, and for deep slides where plane strain deformation conditions may be applicable, possible adjustments in the present procedures for developing p-y relationships appear to warrant further consideration.

GROUP PILE ANALYSES

For illustrative purposes a series of analyses were performed for two simple pile group arrangements, each subjected to two conditions of lateral load. Thirty inch diameter piles were considered and the undrained shear strength of the soil was 100 psf, corresponding to the 30-inch diameter, single pile analyses previously described. Values of 0.3 and 0.10 inch were also assumed for f_c and δ_f , respectively.

The first group arrangement selected is composed of two piles closely spaced at a distance of two pile diameters (60 inches) center to center. The two loading conditions analyzed consist of displacement in directions perpendicular to the centerline connecting the two piles (Case I), and parallel to the centerline (Case II). The soil-pile models for these two cases are illustrated in Fig 15 with the appropriate fixed, symmetry and antisymmetry boundaries indicated. The fixed boundaries shown in this figure were located a distance of 16 pile diameters (480 inches) away from the pile axis. The p-y relationships for both piles in the group are identical and are shown for the two cases of loading in Fig 16, each curve corresponding to the load carried by an individual pile in the group. For comparative purposes the p-y relationship for a single pile analysis is also shown in this figure. Referring to Fig 16, a significant reduction in lateral resistance, as compared to a single pile, is shown to accompany lateral loading in a direction parallel to the centerline of the pile group (Case II). The lateral resistance developed by loading perpendicular to the

center line of the pile group (Case I) is, for the individual piles in the group, roughly equal to the resistance developed by a single pile, although moderately greater displacement is required to develop this resistance in the pile group.

The second group pile arrangement considered consists of a long, "infinite," row of piles spaced a distance of 3 pile diameters (90 inches) apart, center-to-center. The two cases of loading, parallel and perpendicular to the centerline, are identical to those for the closely spaced two pile group described above, and the soil-pile models are illustrated in Fig 17 for this latter pile group arrangement. The single fixed boundary used in these analyses is located a distance of 8 pile diameters (240 inches) from the pile axis. The p-y relationships for all piles in this pile group arrangement are identical and are presented and compared for the two loading cases with the single pile relationship in Fig 18. Again, a significantly lower lateral resistance may be seen to accompany pile displacements parallel to the centerline of the pile row (Case II), while displacement perpendicular to this direction yields lateral resistances comparable to those for a single laterally loaded pile.

CONCLUSIONS

It is essential that in the design of many pile supported offshore platforms special consideration be given to the development of soil resistance-displacement relationships for laterally loaded piles. The procedure presented in this paper employs the finite element method for developing such relationships (p-y curves) for piles in soft, saturated clays subjected to short-term, static loading. This procedure has been found to provide a powerful tool for developing p-y relationships for both single and grouped pile arrangements. For the latter, the procedure provides valuable information on the influence of several important variables including pile spacing, group configuration and direction of pile displacement.

The finite element procedure described appears to yield reasonable numerical results and shows reasonable agreement with the procedure described by Matlock (1970); however, the significant influence of several important variables and characteristics should be emphasized. These include the soil resistance at the soil-pile interface, the stress-strain properties of the soil surrounding the pile, separation of the soil from the pile at very shallow depths, and restricted vertical deformation and confinement of the soil at relatively great depths. The finite element analyses presented herein indicate that significant increases in lateral soil-pile resistance accompany increases in the soil-pile interface

resistance (adhesion), the shear strength and modulus of the soil surrounding the pile, and the degree of vertical confinement of the soil. In addition a decreased resistance may be developed if the soil separates from the pile. Consequently, the accuracy of any numerical procedure for predicting p-y relationships appears largely dependent on the degree to which the soil properties and characterization of the pile behavior represent the actual full-scale field conditions. To reliably establish this degree of correspondence between field behavior and that predicted by the finite element model further comparisons with experimental data and field observations are necessary.

ACKNOWLEDGEMENT

The work described in this paper was partially supported by the National Science Foundation, Grant GK-27824 and the Bureau of Engineering Research at The University of Texas. The writers wish to express their special appreciation to Professors Lymon Reese and Hudson Matlock for their many valuable suggestions during the progress of this work and for their critical review of this paper. Gratitude is also expressed to Mr. R. G. Bea and Shell Oil Company for providing useful data on the soil-pile interface properties.

REFERENCES

1. Bea, R. G., "How Sea-Floor Slides Affect Offshore Structures," The Oil and Gas Journal, Nov. 29, 1971, pp. 88-92.
2. Broms, Bengt B., "Lateral Resistance of Piles in Cohesive Soils," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 90, No. SM2, Proceedings Paper 3825, March, 1964, pp. 27-63.
3. Chuang, John and Reese, Lymon C., "Studies of Shearing Resistance Between Cement Morar and Soil," Research Report No. 89-3, Center for Highway Research, The University of Texas at Austin, October, 1968.
4. Clough, G. W. and Duncan, J. M., "Finite Element Analyses of Port Allen and Old River Locks," Report No. TE-69-3, Geotechnical Engineering, University of California, Berkeley, September, 1969.
5. Doherty, William P., Wilson, Edward L. and Taylor, Robert L., "Stress Analysis of Axisymmetric Solids Utilizing Higher-Order Quadrilateral Finite Elements," Report No. S.E.S.M. 69-3, Structural Engineering Laboratory, University of California, January, 1969.
6. Duncan, James M. and Chang, Chin-Yung, "Nonlinear Analysis of Stress and Strain in Soils," Journal of the Soil Mechanics and Foundations Division, American Society of Civil Engineers, No. SM5, September, 1970, pp. 1629-1651.
7. Goodman, R. E., Taylor, R. L., Brekke, T. L., "A Model for the Mechanics of Jointed Rock," Journal of the Soil Mechanics and Foundations Division, ASCE, No. SM3, May, 1968, pp. 637-659.
8. Konder, Robert L., "Hyperbolic Stress-Strain Response: Cohesive Soils," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 90, No. SM1, February, 1963, pp. 115-143.
9. Ladd, Charles C., "Stress-Strain Modulus of Clay in Undrained Shear," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 90, SM5, September, 1964, pp. 103-132.
10. Matlock, Hudson, "Correlations for Design of Laterally Loaded Piles in Soft Clay," Second Annual Offshore Technology Conference, Houston, Texas, 1970.
11. McClelland, Bramlette and Focht, John A., Jr., "Soil Modulus for Laterally Loaded Piles," Transactions, ASCE, Vol. 123, New York, 1958, pp. 1049-1063.
12. Potyondy, J. G., "Skin Friction Between Various Soils and Construction Materials," Geotechnique, Vol. 11, No. 4, London, December, 1961.
13. Skempton, A. W., "The Bearing Capacity of Clays," Building Research Congress, Division 1, Part 3, London, 1951, pp. 180-189.
14. Wright, Stephen G. and Dunham, Robert S., "Bottom Stability Under Wave Induced Loading," Preprint Vol. I, Fourth Annual Offshore Technology Conference, Houston, Texas, 1972, pp. 853-862.

TABLE 1 - ULTIMATE SOIL RESISTANCE COMPUTED FOR ASSIGNED INTERFACE PROPERTIES

f_c	=	Infinite	1.0	0.6	0.6	0.3	0.3	0
δ_f (inch)	=	-	0.10	0.02	0.10	0.02	0.10	-
P_u (lb/in)	=	175	167	154	153	141	141	128

TABLE 2 - SOIL PROPERTIES FOR COMPARATIVE ANALYSIS WITH 24-IN. DIAMETER PILE

Property	Depth, 4D (8 feet)	Depth, 16D (32 feet)
Strength, s	160 psf	180 psf
G_i	27,600 psf	51,800 psf
γ_f	11 percent	21 percent

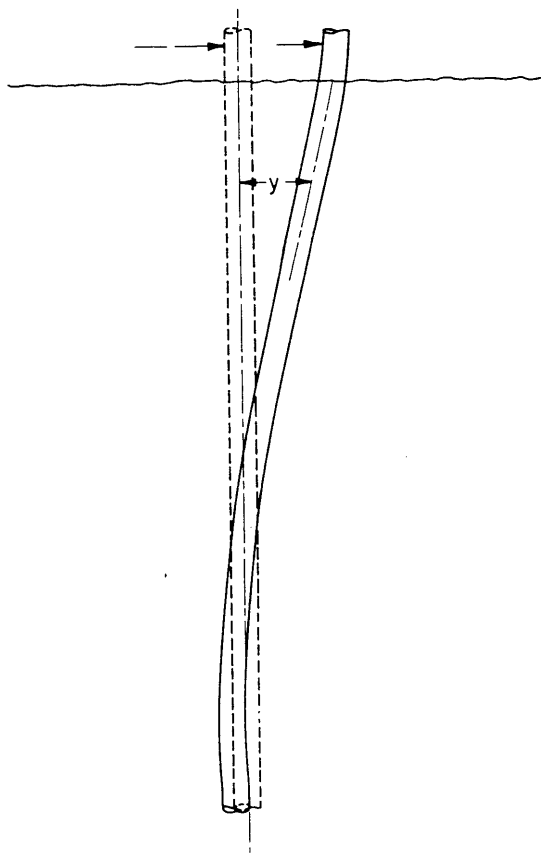


Fig. 1 - Typical laterally loaded pile.

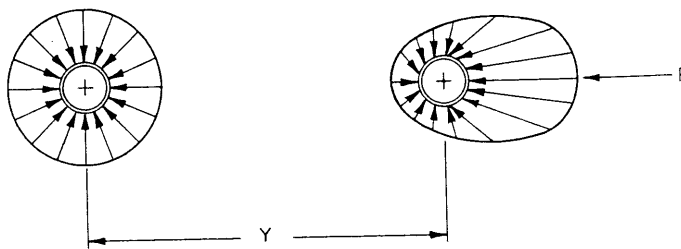


Fig. 2A - Initial stress distribution around pile cross-section.

Fig. 2B - Equilibrium stress distribution around laterally displaced pile cross-section.

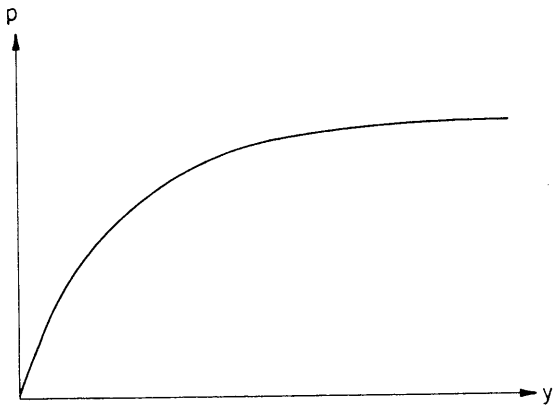


Fig. 3 - Typical p-y relationship.

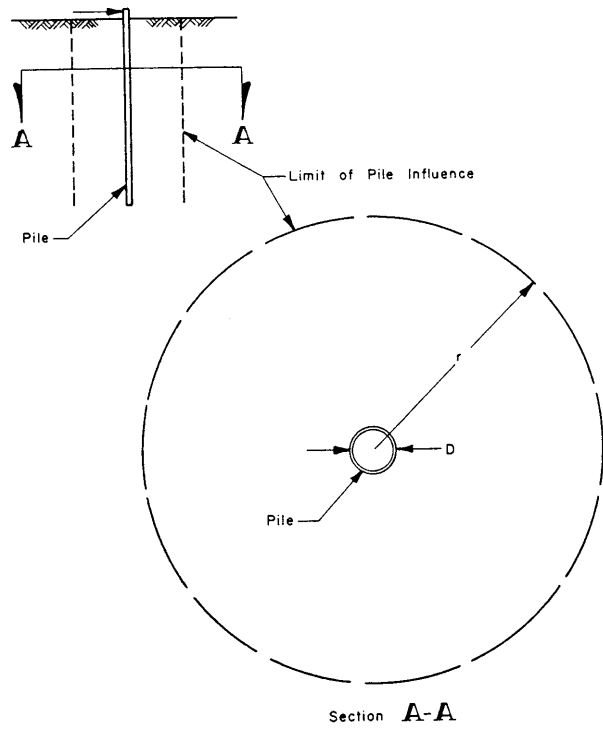


Fig. 4 - Zone of pile influence for laterally loaded pile.

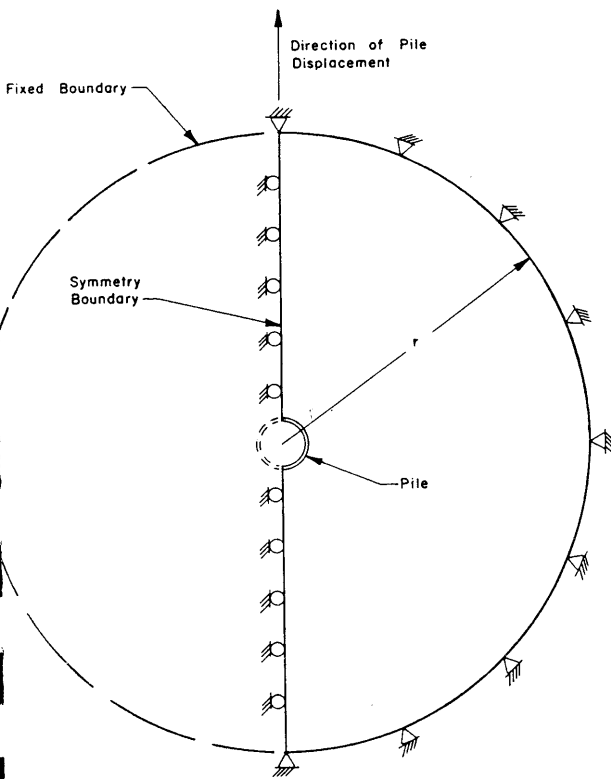


Fig. 5 - Symmetry boundary for soil-pile cross-section subjected to lateral displacement.

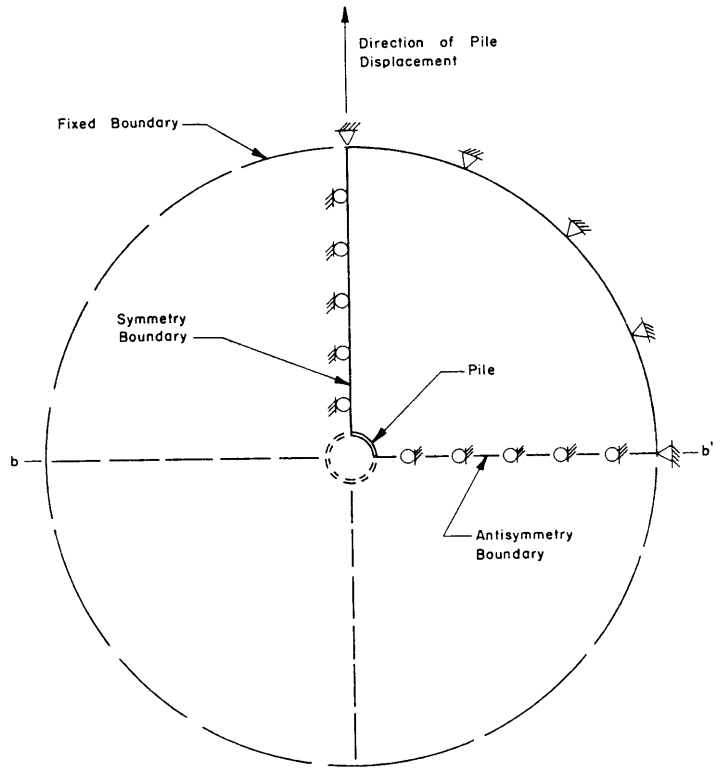


Fig. 6 - Antisymmetry and symmetry boundaries for soil-pile cross-section.

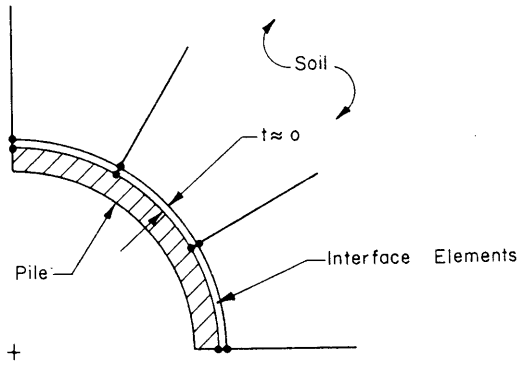


Fig. 7A - Interface element at soil-pile boundary.

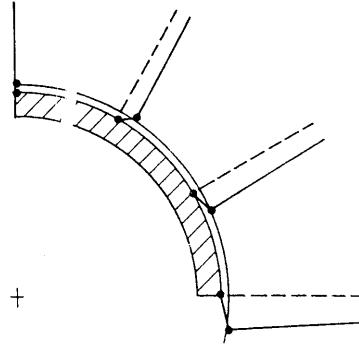


Fig. 7B - Displacement pattern for interface elements.

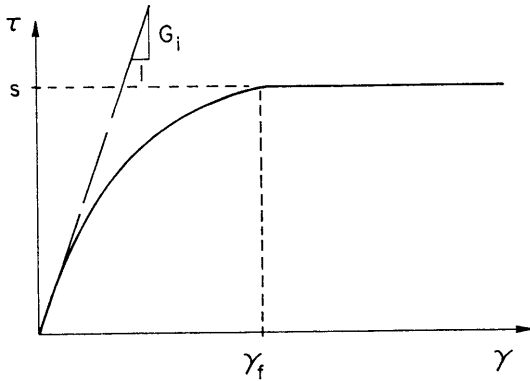


Fig. 8A - Hyperbolic characterization of maximum shear stress-strain relationship.

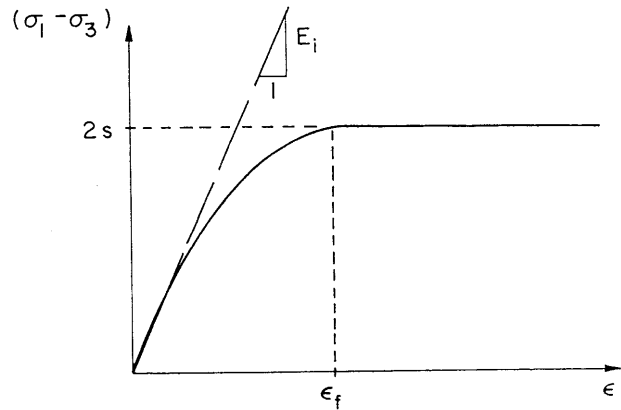


Fig. 8B - Hyperbolic characterization of axial stress-strain relationship for the triaxial test.

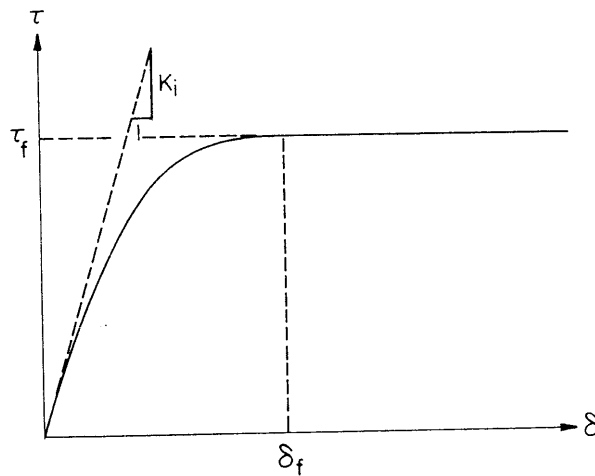


Fig. 9 - Hyperbolic shear stress-relative displacement relationship for soil-pile interface elements.

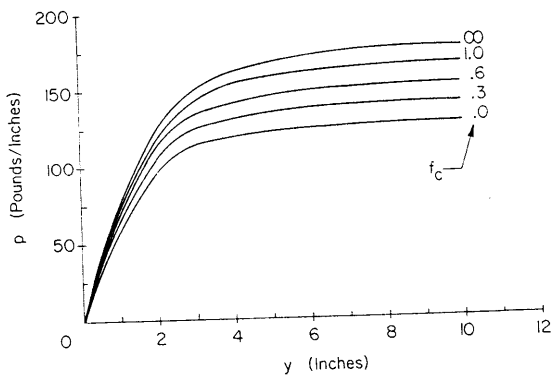


Fig. 10 - Influence of interface resistance (adhesion factor, f_c) on the p-y relationships for a laterally loaded pile.

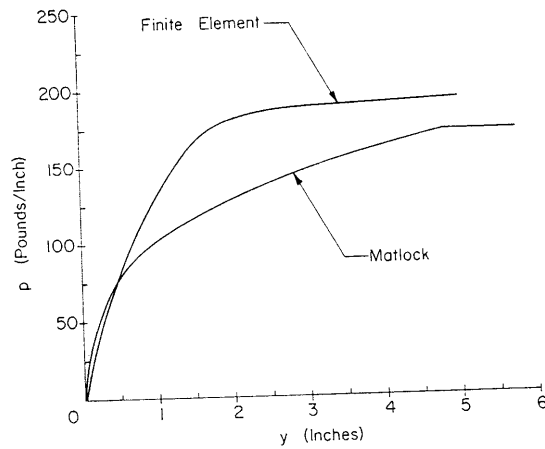


Fig. 11 - The p-y relationships for 24-in. diameter "shallow" pile comparison, depth = 4 D.

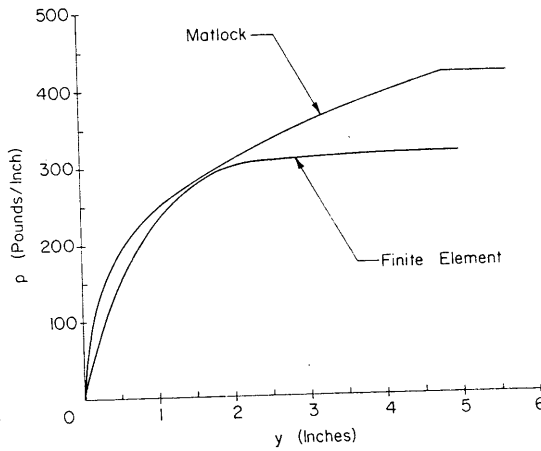


Fig. 12 - The p-y relationships for 24-in. diameter "deep" pile comparison, depth = 16 D.

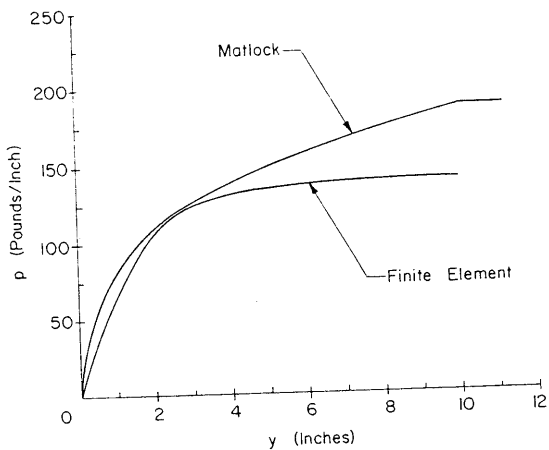


Fig. 13 - The p-y relationships for 30-in. diameter "deep" pile comparison, depth = 8 D.

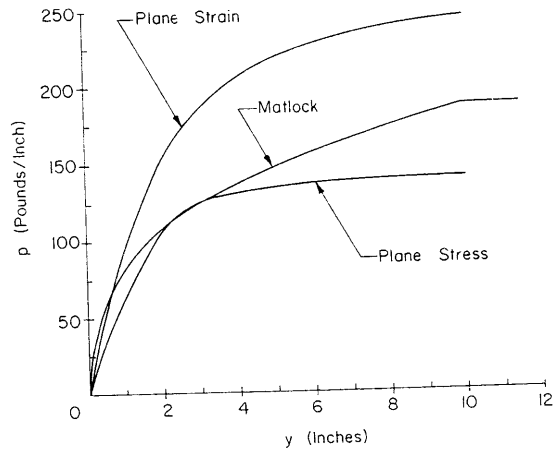


Fig. 14 - Comparison of Matlock p-y relationship with plane stress and plane strain finite element analyses, 30-in. diameter pile, depth = 8 D.

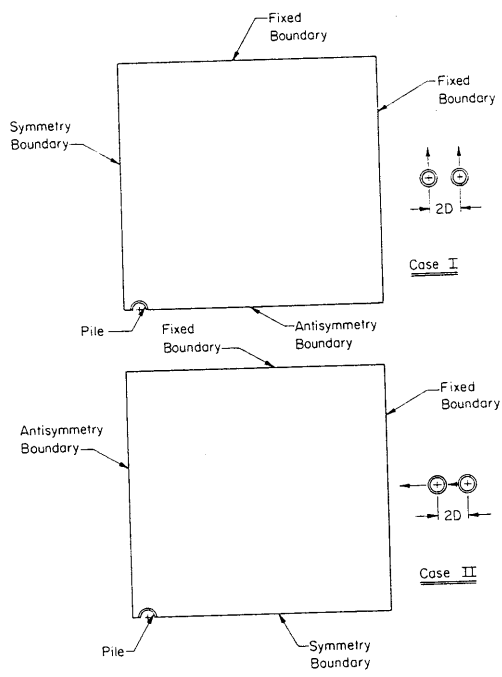


Fig. 15 - Soil-pile models for two pile group.

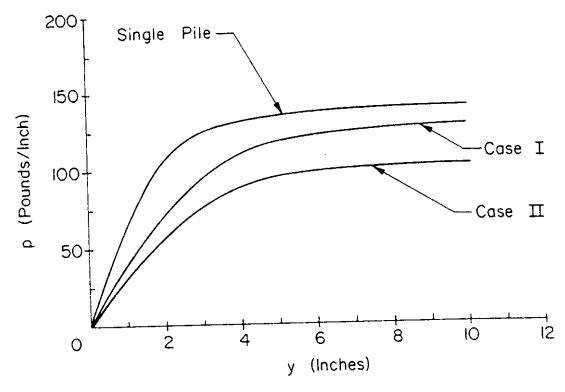


Fig. 16 - The p-y relationships for two pile group and single pile analyses, group pile spacing = $2D$.

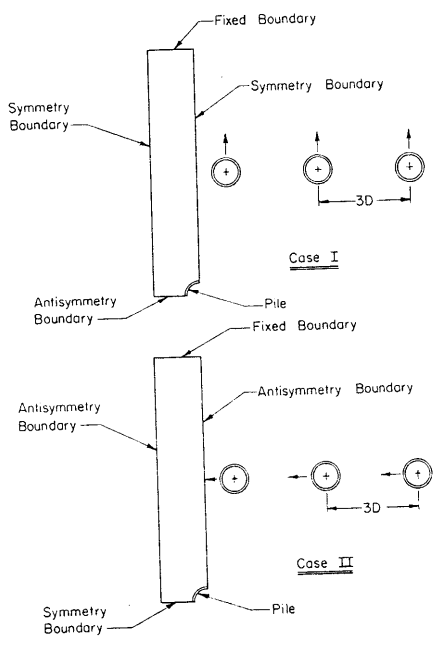


Fig. 17 - Soil-pile models for "infinite" pile row.

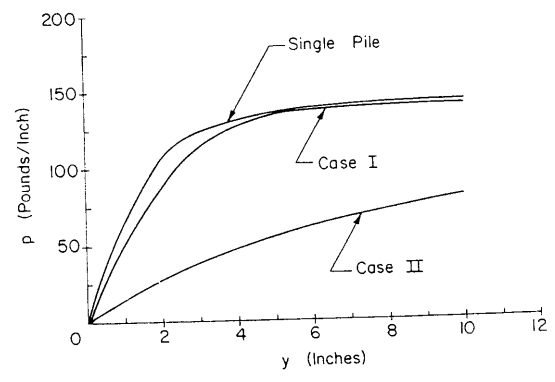


Fig. 18 - The p-y relationships for "infinite" pile row and single pile analyses, group pile spacing = $3D$.