

Liquefaction Response of Partially Saturated Sands.

II: Empirical Model

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Abstract: Partial saturation in sands attributable to the presence of gas bubbles (not capillarity) can be encountered naturally in the field because of the decomposition of organic matter, or it can be induced for liquefaction mitigation. An empirical model (RuPSS) was developed to predict the excess pore pressure ratio (r_u) in partially saturated sands subjected to earthquake-induced shear strains. The model is based on experimental test results on partially saturated sands. Cyclic simple shear strain tests were performed on specimens prepared and tested in a special liquefaction box. Excess pore pressures were measured for a range of degrees of saturation $40\% < S < 90\%$, relative densities $D_r = 20 - 67\%$, and cyclic shear strains $\gamma = 0.01 - 0.2\%$. The test results demonstrated that partially saturated sands achieved a maximum excess pore pressure ratio ($r_{u,max}$) when large enough cycles of shear strain were applied. The excess pore pressure ratio (r_u) that partially saturated sand can achieve under a given earthquake-induced peak shear strain and the number of equivalent cycles of application can be significantly smaller than $r_{u,max}$. Therefore, the empirical model was developed in two stages. In the first stage, $r_{u,max}$ was related to S , D_r , and shear strain (γ). In the second stage, a model was developed relating r_u to $r_{u,max}$, shear strain amplitude (γ), effective stress (σ'_v), and earthquake magnitude (M). This paper presents the equations that define the predictive models for $r_{u,max}$ and r_u . Through these equations, plots for $r_{u,max}$ and r_u are provided for ranges of soil and earthquake parameters for ease of use in engineering applications. To illustrate the implementation of the empirical model for predicting $r_{u,max}$ and r_u , an example is presented in which a partially saturated sand layer experiencing a peak earthquake-induced shear strain was analyzed, and the pore pressure response of the sand was evaluated using both the predictive equations and the plots. DOI: 10.1061/(ASCE)GT.1943-5606.0000816. © 2013 American Society of Civil Engineers.

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Introduction

Understanding the liquefaction response of partially saturated loose sands has been receiving increased attention in the geotechnical earthquake engineering field. Partially saturated sands, where a reduction in the degree of saturation is attributable to the presence of gas bubbles and not attributable to capillarity, can be encountered naturally as a result of biological activities (Wheeler 1988; Mitchell and Santamarina 2005), or it can be induced as a means of liquefaction mitigation as suggested by Okamura et al. (2006), Yegian et al. (2007), and U.S. Patent No. 7,192,221.

The resistance of partially saturated sands to liquefaction has been investigated by a number of researchers (Chaney 1978; Yoshimi et al.

1989; Tsukamoto et al. 2002; Ishihara and Tsukamoto 2004; Yang et al. 2004; Okamura et al. 2006, 2011). In all of these studies, the liquefaction criterion was based on a specimen reaching 5% double-amplitude (DA) axial strain under constant cyclic stresses. Hence, liquefaction resistance of partially saturated sands was correlated to an increasing number of cycles to reach 5% DA axial strain for the same cyclic stress applied. Also, Ishihara and Tsukamoto (2004) expressed resistance against liquefaction in terms of an increased factor of safety (FS).

The authors have conducted experimental investigations to evaluate liquefaction response of partially saturated sands in terms of excess pore pressure generation under cyclic simple shear strains. Dobry et al. (1982) demonstrated that excess pore pressure generation in saturated sands is more related to cyclic strains induced by an earthquake rather than cyclic stresses.

In a companion paper, the results from a series of cyclic simple shear strain tests are presented and discussed (Eseller-Bayat et al. 2013). The tests were conducted using a special liquefaction box and a shaking table to investigate excess pore pressure ratio (r_u) generation in partially saturated sands. This paper presents the formulations of an empirical model (RuPSS) for r_u prediction in partially saturated sands subjected to earthquake-induced shear strains. The model is based on the test results presented in the companion paper.

Model Framework

Fig. 1 shows typical excess pore pressure ratio generation in a partially saturated specimen as a function of the number of strain

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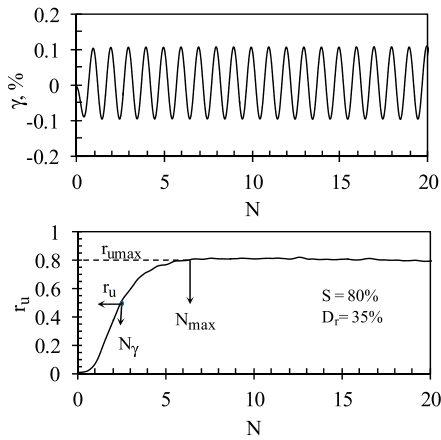


Fig. 1. Typical excess pore pressure ratio generation in a partially saturated specimen during a cyclic simple shear strain test ($\sigma'_v = 2.5$ kPa)

cycles (N). The parameters of interest in the formulation of the predictive model are indicated on the plots, namely, degree of saturation (S), relative density (D_r), cyclic shear strain amplitude (γ), vertical effective stress (σ'_v), maximum excess pore pressure ratio ($r_{u,max}$), number of shear strain cycles at which $r_{u,max}$ is achieved (N_{max}), excess pore pressure ratio (r_u), and number of equivalent shear strain cycles associated with an earthquake strain time history (N_γ).

The ultimate goal of the development of the model was to provide a means for predicting excess pore pressure ratios (r_u) in partially saturated sands subjected to seismic shear strains. Fig. 2 depicts a soil profile of partially saturated sand experiencing ground motions associated with an input acceleration record of an earthquake with a magnitude M . The output of the ground motion analysis at a given depth can be expressed in terms of a shear strain time history with a peak shear strain amplitude of γ_{max} . To make use of the experimental results from cyclic shear strain tests, this maximum shear strain has to be converted to an equivalent cyclic shear strain in a manner similar to the procedure followed for fully saturated sands. The program *SHAKE91* has presented the concept of strain ratio as $R = \gamma/\gamma_{max}$, where R can be expressed in terms of the earthquake magnitude M [$R = (M - 1)/10$]. With the strain ratio R , the earthquake-induced strain time history can be converted to an equivalent number and amplitude of shear strain cycles (N_γ, γ). The excess pore pressure ratio (r_u) then can be predicted using the empirical model developed based on the experimental test results.

The number of equivalent shear strain cycles (N_γ) can be estimated either from the shear strain time history obtained from a ground motion analysis or using empirical data. In this research, N_γ was related to R and M using the data of Seed et al. (1975) and Wer and Dobry (1982).

The development of the r_u predictive model (RuPSS) for partially saturated sands was achieved in two stages.

1. The function f_1 given in Eq. (1) was established, relating $r_{u,max}$ to the degree of saturation (S), relative density (D_r), and equivalent cyclic shear strain ($\gamma = \gamma_{max} \times R$)

$$r_{u,max} = f_1(S, D_r, \gamma) \quad (1)$$

2. The function f_2 given in Eq. (2) was established, relating r_u [excess pore pressure ratio achieved during a given seismic event generating N_γ equivalent shear strain cycles (γ)] to $r_{u,max}$

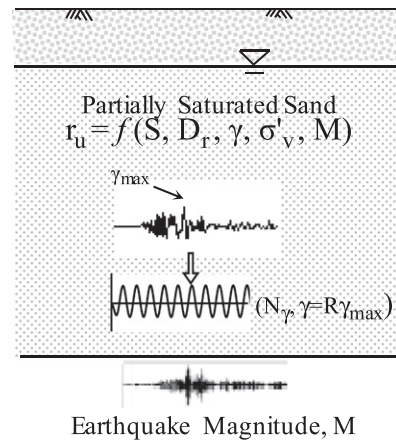


Fig. 2. Excess pore pressure ratio r_u generated in a partially saturated sand profile experiencing an earthquake excitation

(maximum excess pore pressure ratio that can be achieved if N_{max} shear strain cycles are applied)

$$\frac{r_u}{r_{u,max}} = f_2\left(\frac{N_\gamma}{N_{max}}\right) \quad (2)$$

As will be demonstrated later, N_γ can be related to R and M , while N_{max} can be expressed in terms of $r_{u,max}$, γ , and σ'_v .

Finally, combining the two previously mentioned functions, the final function (f) that provides an estimate of r_u in partially saturated sand subjected to a seismic excitation was established as shown in Eq. (3)

$$r_u = f_1 \times f_2 = f(S, D_r, \gamma, \sigma'_v, M) \quad (3)$$

Typical Results from Cyclic Shear Strain Tests on Partially Saturated Sands

To investigate the effects of the parameters S , D_r , γ , and σ'_v on r_u generation, cyclic shear strain tests were performed using a special liquefaction box (CSSLB) and the loading mechanism of a shaking table. The details of the test setup and results are presented in a companion paper. A total of 96 tests were performed, where the results of the initial 24 tests were used to develop a preliminary understanding of the behavior of partially saturated sands and to plan the details for the additional tests. Eventually, the entire set of data was used to develop the predictive model (RuPSS) for r_u . The influence of each parameter on r_u was investigated, and the observations were used to guide the development of the predictive model. Figs. 3–5 show typical test results from which the following trends are observed.

1. For a given relative density $D_r = 35 - 40\%$ and a shear strain amplitude $\gamma = 0.1\%$, as the degree of saturation (S) is reduced, the excess pore water pressure ratio (r_u) decreases [Fig. 3(b)]. The lower the degree of saturation, the smaller is the $r_{u,max}$ and the larger is the N_{max} . While at $S = 100\%$, $r_{u,max}$ is 1.0; for $S < 90\%$ based on data from this research and for $S = 96.3\%$ based on data published earlier by the authors (Yegian et al. 2007), $r_{u,max}$ is always smaller than 1, indicating that partially saturated sands $S < 96.3\%$ do not achieve initial liquefaction ($r_{u,max} = 1$).
2. Test results shown in Fig. 3(c) demonstrate that relative density has a significant influence on the rate of generation

- of excess pore pressure. The denser the sand, the slower is the rate of increase in r_u . Also, as the density of the sand increases, $r_{u,max}$ decreases and N_{max} increases.
- The experimental results shown in Fig. 4 demonstrate that, as the shear strain amplitude increases, $r_{u,max}$ increases and the number of cycles required to reach $r_{u,max}$ (N_{max}) decreases. The effect of shear strain amplitude on $r_{u,max}$ is smaller than that of S and D_r .
 - The effect of σ'_v on $r_{u,max}$ was also investigated. Under the constraints of the experimental setup, $r_{u,max}$ values were measured for σ'_v between 1.44 and 9.86 kPa. Fig. 5 shows 21 data points of $r_{u,max}$ normalized with respect to $r_{u,max}$ at $\sigma'_v = 2.4$ kPa. The results show that initial effective stress (within the range tested) has little influence on $r_{u,max}$. The

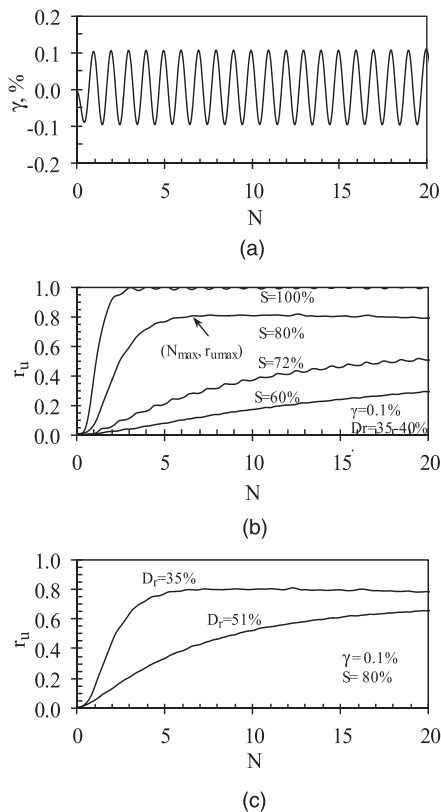


Fig. 3. (a) Typical cyclic shear strain record; (b) effect of degree of saturation (S) on excess pore pressure ratio (r_u); (c) effect of relative density (D_r) on r_u ($\sigma'_v = 2.5$ kPa)

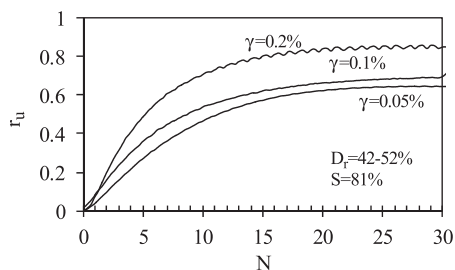


Fig. 4. Effect of cyclic shear strain amplitude (γ) on excess pore pressure ratio r_u in partially saturated sand specimen with $S = 81\%$ ($\sigma'_v = 2.5$ kPa)

slight variability in the data is attributable to the accuracy and reproducibility of the tests.

These and other observations from the experimental test results were used to develop the model (RuPSS) for predicting the excess pore pressure ratio (r_u) in partially saturated sands during earthquakes.

Prediction of Maximum Excess Pore Pressure Ratio

If a partially saturated sand specimen with a certain relative density is subjected to a cyclic shear strain amplitude of γ , then after a certain number of cycles (N_{max}), the excess pore pressure ratio will reach a maximum value of $r_u = r_{u,max}$. As was described earlier, test results on partially saturated specimens showed that $r_{u,max}$ depends significantly on the degree of saturation (S) and to a lesser extent on the relative density (D_r) and the amplitude of the cyclic shear strain (γ). Fig. 6 shows further test results confirming that S has a more important influence on $r_{u,max}$ than D_r . Because S is observed to be the most dominant parameter, the formulation of a model for predicting $r_{u,max}$ was first based on establishing an equation that related $r_{u,max}$ to only S for a sand in its loosest condition ($D_r = 20\%$) and for a shear strain amplitude of $\gamma = 0.1\%$. This equation is referred to as the base function f_b . A scaling factor function F_D was then established to relate $r_{u,max}$ generated at $D_r = 20\%$ to relative densities greater than 20% . Similarly, a scaling factor function F_γ was developed to relate $r_{u,max}$ generated at a shear strain of 0.1% to other levels of shear strain amplitudes. The final $r_{u,max}$ model function f_1 [Eq. (1)] was obtained by the product of the base function f_b and the scaling factor functions F_D and F_γ as in Eq. (4)

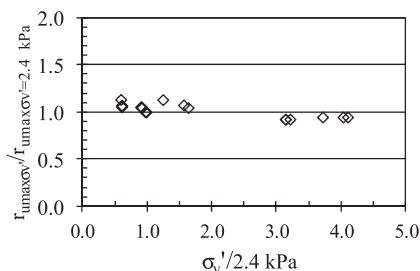


Fig. 5. Effect of initial effective stress (σ'_v) on maximum excess pore pressure ratio ($r_{u,max}$)

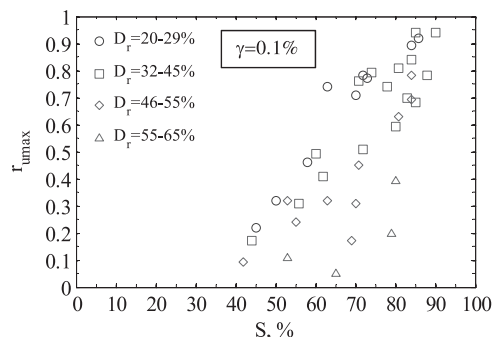


Fig. 6. Laboratory data of $r_{u,max}$ for different degrees of saturation (S) and relative densities (D_r) and cyclic shear strain amplitude γ of 0.1%

$$r_{u,\max} = f_1(S, D_r, \gamma) = f_b(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_\gamma(S, \gamma) \quad (4)$$

The base function f_b and the scale factor functions F_D and F_γ were established ultimately using all 96 data points on partially saturated sand specimens with parameters $S = 40 - 90\%$, $D_r = 20 - 67\%$, and $\gamma = 0.01 - 0.2\%$. Details of the formulations of these functions and estimations of model parameters and their statistics are presented in Eseller-Bayat (2009). The results of these analyses led to the following functions that relate $r_{u,\max}$ to S , D_r , and γ :

$$f_b = S^{0.5} \times \exp\left[-\left(\frac{1-S}{0.54}\right)^4\right] \quad (5)$$

$$F_D = 1 - 8.75 \times (D_r - 0.2) \times (1 - S) \times \exp\left\{-\frac{(1-S)^2}{2 \times [1 - 0.84 \times (0.2/D_r)^{0.25}]^2}\right\} \quad (6)$$

$$F_\gamma = 1 - 1.75 \times \left(-\log\frac{\gamma}{0.001}\right) \times (1 - S) \times \exp[-3.1(1 - S)^2] \quad (7)$$

Note that $F_D = 1$ for $D_r = 20\%$ and that $F_\gamma = 1$ for $\gamma = 0.1\%$.

Model adequacy or the goodness of fit was evaluated by calculating the mean square error ($MS = 0.007$) and the coefficient of determination ($R^2 = 0.92$) for all 96 test data. The low mean square error and the high coefficient of determination demonstrate that the predicted $r_{u,\max}$ values from the model are in good agreement with the experimental data. In Figs. 7 (a-d), comparisons are made between the experimental data and the $r_{u,\max}$ values predicted by the model shown in Eqs. (4)–(7). Because of the complexity of the equations of the predictive model, for ease of estimation, two plots of $r_{u,\max}$ were generated for partially saturated loose ($D_r = 25\%$) and medium-dense ($D_r = 50\%$) sands, for varying levels of shear strain, as shown in Figs. 8(a and b). It is noted that these plots provide estimates of $r_{u,\max}$, assuming that the sand is subjected to enough shear strain cycles (N_{\max}) with amplitude γ to achieve $r_{u,\max}$. If the number of equivalent cycles of a seismic shear strain history is fewer than N_{\max} , then the excess pore pressure ratio (r_u) will be smaller than $r_{u,\max}$. In the section Prediction of Excess Pore Pressure Ratio (r_u), a predictive model for r_u is presented.

Prediction of Excess Pore Pressure Ratio

In the section Prediction of Maximum Excess Pore Pressure Ratio, an empirical model (f_1) was presented that can be used to predict the maximum excess pore pressure ratio ($r_{u,\max}$) in partially saturated sands. The model assumes that the number of applied shear strain cycles is large enough to achieve the maximum value of the excess

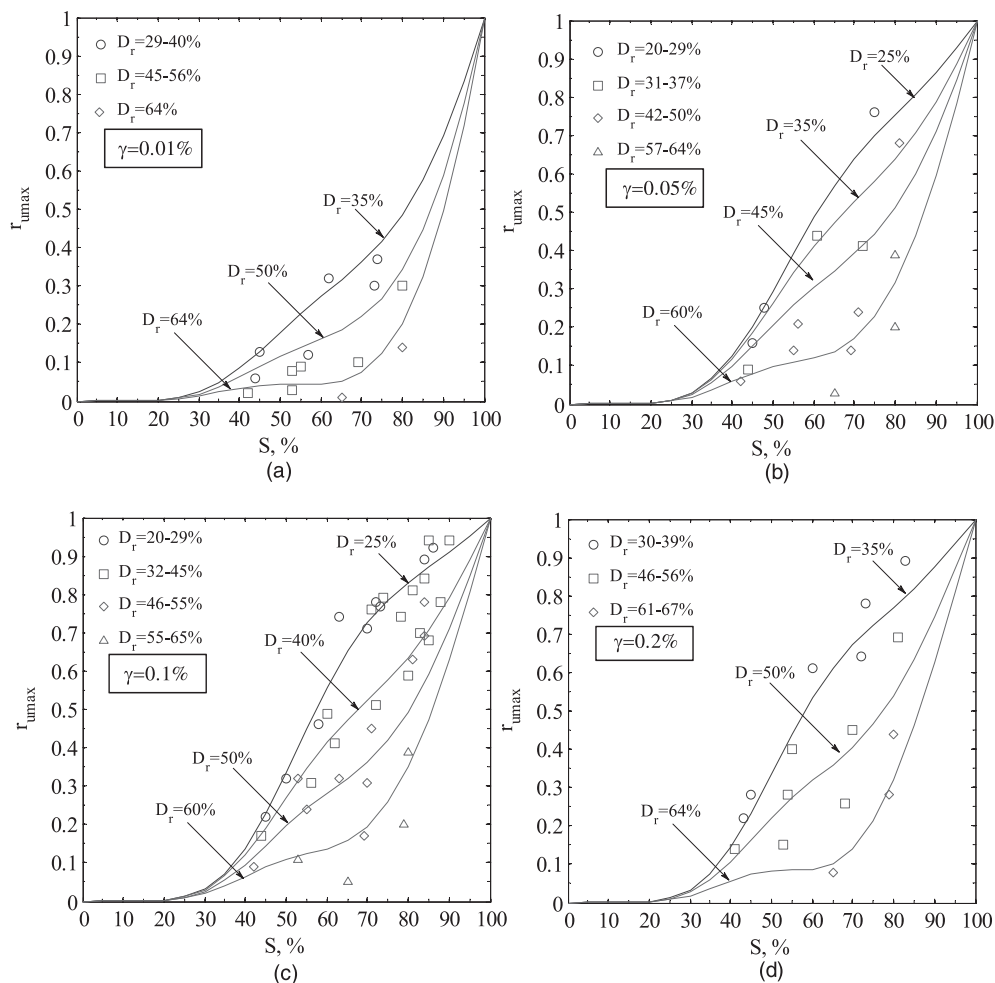


Fig. 7. Comparisons of $r_{u,\max}$ from laboratory data and model predictions

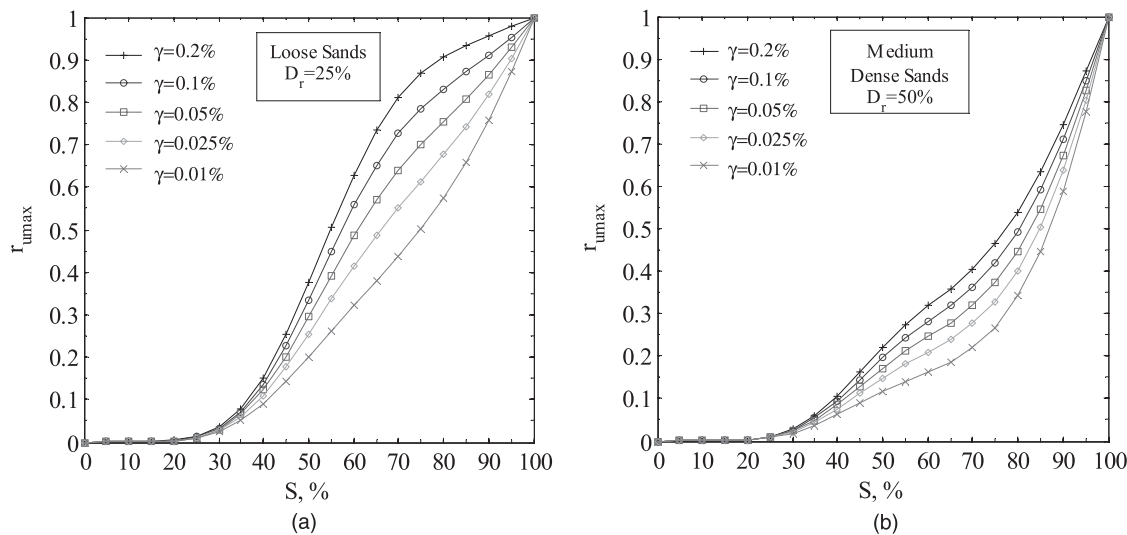


Fig. 8. Model predictions of maximum excess pore pressure ratio ($r_{u,max}$) in (a) loose ($D_r = 25\%$) and (b) medium-dense ($D_r = 50\%$) sands

pore pressure ratio. However, earthquakes with different magnitudes will apply different numbers of equivalent shear strain cycles (N_γ). Hence, if the magnitude of a design event is small enough that N_γ is less than N_{max} , then r_u will be less than $r_{u,max}$ (Fig. 1). To evaluate the rate of increase of r_u with the number of shear strain cycles, the test results from r_u were normalized with $r_{u,max}$ and plotted versus N/N_{max} , as shown in Fig. 9. The trends observed in the plots for partially saturated sands are generally similar to those for fully saturated sands as demonstrated by Chang et al. (2007). The slight variation in the rates of excess pore pressure generation could be because of secondary influences of S , D_r , and γ beyond what is shown in Eq. (1).

To establish a model for the estimation of r_u , a function f_2 was established using the normalized $r_u/r_{u,max}$ versus N/N_{max} plots of Fig. 9. As defined earlier, N_γ is the number of equivalent shear strain cycles associated with a seismic event, and hence, N_γ can be used as N in Fig. 9 to obtain $r_u/r_{u,max}$. The trigonometric function shown in Eq. (8) was determined to be best suited to describe the trends observed in the data shown in Fig. 9

$$\frac{r_u}{r_{u,max}} = f_2\left(\frac{N_\gamma}{N_{max}}\right) = \left\{ \frac{\sin\left[\left(\frac{N_\gamma}{N_{max}} - 0.5\right) \times \pi\right] + 1}{2} \right\}^\theta$$

for $N_\gamma/N_{max} \leq 1$

$$\frac{r_u}{r_{u,max}} = 1 \quad \text{for } N_\gamma/N_{max} > 1 \quad (8)$$

Based on a statistical analysis of the data, upper bound (95% prediction limit), median, and lower bound (5% prediction limit) functions were established, resulting in values of the parameter θ in Eq. (8) of 0.25, 0.54, and 1.1, respectively. Fig. 9 includes these limit lines.

The number of equivalent shear strain cycles (N_γ) can be obtained either using the shear strain record computed through a ground motion analysis of the soil profile in a way similar to that followed for fully saturated sand or empirically by using the strain ratio R and earthquake magnitude M .

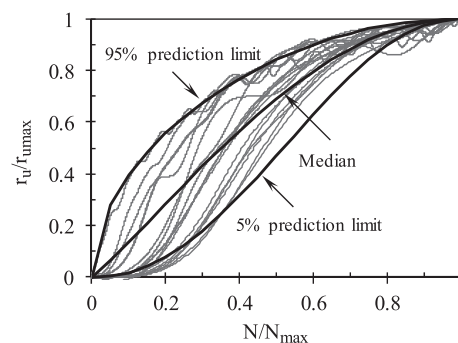


Fig. 9. Normalized excess pore pressure ratio ($r_u/r_{u,max}$) versus normalized number of cyclic shear strain (N/N_{max})

The predictive model for r_u presented in this paper uses empirically estimated N_γ . The development of the procedure for estimating N_γ involved relating the number of equivalent shear strain cycles for $R = 0.65$ to M using the data from Seed et al. (1975). Based on a regression analysis of the data, the following relationship was established:

$$N_\gamma(R = 0.65) = 0.057 \exp(0.72M) \quad (9)$$

To estimate N_γ for any R value, the data of Wer and Dobry (1982) was used, relating $N_\gamma(R = R)$ to $N_\gamma(R = 0.65)$ as shown in Eq. (10)

$$\frac{N_\gamma(R = R)}{N_\gamma(R = 0.65)} = 0.114 \times \exp\left[\left(\frac{1}{R}\right)^{1.8}\right] \quad (10)$$

Combining Eqs. (9) and (10) leads to

$$N_\gamma(R = R) = 0.114 \times \exp\left[\left(\frac{1}{R}\right)^{1.8}\right] \times 0.057 \exp(0.72M) \quad (11)$$

If the expression of *SHAKE91* is used to relate R to M [$R = (M - 1)/10$], then Eq. (11) can be expressed solely in terms of the earthquake magnitude (M) as in Eq. (12)

$$N_\gamma = 0.0065 \times \exp \left[\left(\frac{10}{M-1} \right)^{1.8} + 0.72M \right] \quad (12)$$

It is noted that N_γ , as defined herein, is the number of equivalent shear strain cycles and is different from the number of uniform stress cycles as was defined by Seed et al. (1975). Seed's number of uniform stress cycles was developed for a stress ratio of $R = 0.65$. For a constant stress ratio R , the larger the magnitude, the larger is the number of uniform stress cycles. However, when R as defined by SHAKE91 [$R = (M - 1)/10$] is adjusted for magnitude, the larger the magnitude, the larger is R , leading to a fewer number of equivalent shear strain cycles, as shown in Fig. 10. The slight increase in N_γ for magnitudes larger than 7.0 is attributable to the nature of Eqs. (9) and (10) and that of R developed by various investigators.

The number of shear strain cycles at which $r_{u,max}$ is achieved (N_{max}) was observed to be dependent on S , D_r , γ , and σ'_v . Because $r_{u,max}$ incorporates the effects of S , D_r , and γ , it was decided to relate N_{max} to $r_{u,max}$ and σ'_v . Because partially saturated specimens were tested under relatively small $\sigma'_v = 2.5$ kPa, the effect of larger σ'_v on N_{max} was introduced in a similar way to that on the number of cycles to liquefaction (N_L) in fully saturated sands (Chang et al. 2007; Hazirbaba 2005; Dobry et al. 1982). Details of the formulations can be found in Eseller-Bayat (2009). Eq. (13) shows the expression that relates N_{max} to $r_{u,max}$, γ , and σ'_v

$$N_{max} = 107 \times \exp[-(3r_{u,max} + 2,011\gamma)] \left(\frac{1}{\text{kPa}} \right) \times \sigma'_v \quad (13)$$

σ'_v is in kPa

In summary, r_u can be estimated following these three steps:

1. Compute the maximum excess pore pressure ratio ($r_{u,max}$) from function f_1 using Eqs. (4)–(7).
Alternatively, the plots in Fig. 8 for loose and medium-dense sands can be used to estimate $r_{u,max}$ for $D_r = 25\%$ and 50% . For other D_r values, linear interpolation between the two plots is suggested. These plots were generated using Eqs. (4)–(7).
2. Compute $r_u/r_{u,max}$ for a given earthquake event from function f_2 using Eqs. (8)–(13).
3. Compute the excess pore pressure ratio (r_u) from the function $f = f_1 \times f_2$.

It is noted that the empirical model, while using measured pore pressure results from laboratory tests on partially saturated sands, also utilizes empirical correlations that relate the earthquake

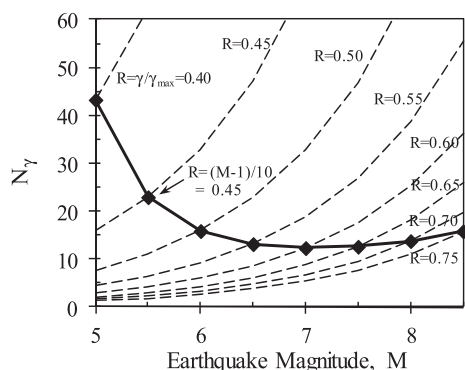


Fig. 10. Number of equivalent shear strain cycles (N_γ) as a function of earthquake magnitude (M) and strain ratio (R)

magnitude to the stress ratio R and the number of equivalent shear strain cycles. Sensitivity analyses of r_u computed using the empirical model as a function of various model parameters including the earthquake magnitude led to the conclusion that the model is most reliable for magnitudes greater than 6, a magnitude range of particular concern with regards to liquefaction. For magnitudes smaller than 6, the predicted number of equivalent shear strain cycles is found to be too large.

Alternatively, Fig. 11 can be used for a conservative estimate of r_u . The plots were generated using Eq. (8) with $\theta = 0.25$ and Eqs. (12) and (13). It is noted that M has little effect on the r_u plots, because it has little effect on N_γ , for $M > 6$. The larger the M , the larger is the R , thus compensating for the effect of M on N_γ . However, σ'_v has an appreciable effect on r_u . The plots are generated using $\sigma'_v = 50$ kPa. Under larger σ'_v , the r_u values will be smaller than what the plots in Fig. 11 indicate.

Example Application of RuPSS Model

To illustrate the steps involved in estimating the earthquake-induced excess pore pressure ratio (r_u) in a partially saturated sand layer where partial saturation is naturally occurring or induced for liquefaction mitigation, the following example is presented. The soil and the ground motion parameters are shown in Fig. 12.

The two-step procedure summarized in the section Prediction of Excess Pore Pressure Ratio (r_u) is implemented as follows.

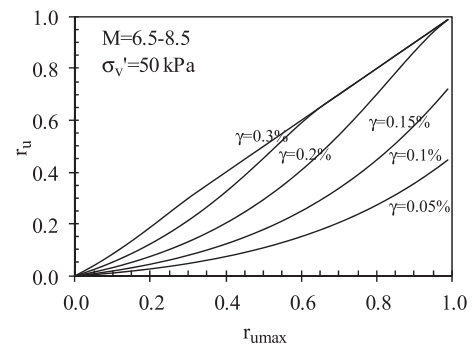


Fig. 11. Conservative estimates of excess pore pressure ratio r_u in partially saturated sands

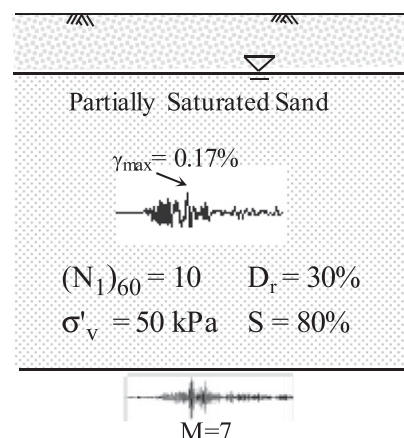


Fig. 12. Illustrative example of excess pore pressure ratio prediction in partially saturated sands using RuPSS model

Step 1: Compute $r_{u,max}$

Start with

$$R = \frac{(7-1)}{10} = 0.6\gamma = 0.6 \times 0.0017 = 0.1\%$$

From Eqs. (4)–(7)

$$r_{u,max} = f_b(S, D_r = 20\%, \gamma = 0.1\%) \times F_D(S, D_r) \times F_\gamma(S, \gamma)$$

$$f_b = 0.8^{0.5} \times \exp\left[-\left(\frac{1-0.8}{0.54}\right)^4\right] = 0.878$$

$$F_D = 1 - 8.75 \times (0.3 - 0.2) \times (1 - 0.8)$$

$$\times \exp\left\{-\frac{(1-0.8)^2}{2 \times [1 - 0.84 \times (0.2/0.3)^{0.25}]^2}\right\} = 0.876$$

$$F_\gamma = 1 - 1.75 \times \left(-\log\frac{0.001}{0.001}\right) \times (1 - 0.8)$$

$$\times \exp\left\{-\left[3.1(1-0.8)^2\right]\right\} = 1.0$$

$$r_{u,max} = 0.878 \times 0.876 \times 1.0 = 0.77$$

Alternatively, from Fig. 8 for $D_r = 25\%$, $r_{u,max} = 0.83$, and for $D_r = 50\%$, $r_{u,max} = 0.49$. For $D_r = 30\%$, when interpolated between $D_r = 25$ and 50% plots, $r_{u,max} = 0.76$, which is in good agreement with the value computed from Eqs. (4)–(7).

Step 2: Compute $r_u/r_{u,max}$

From Eqs. (8), (11), and (12)

$$N_\gamma = 0.0065 \times \exp\left\{\left[\left(\frac{10}{7-1}\right)^{1.8}\right] + 0.72(7)\right\} \cong 12 \text{ cycles}$$

$$N_{max} = 107 \times \exp\{-[3(0.77) + 2,011(0.001)]\} \left(\frac{1}{\text{kPa}}\right) \\ \times 50 \cong 71 \text{ cycles}$$

$$\frac{r_u}{r_{u,max}} = \left\{\frac{\sin[(12/71 - 0.5) \times \pi] + 1}{2}\right\}^\theta$$

With $\theta = 0.25$ and 0.54 , the upper bound and average values of $r_u/r_{u,max}$ are 0.51 and 0.24 .

Step 3 Compute r_u

From Eq. (3)

$$r_u = r_{u,max} \left(\frac{r_u}{r_{u,max}}\right)$$

$$r_u = 0.77 \times 0.51 = 0.4 \quad (\text{upper bound})$$

$$r_u = 0.77 \times 0.24 = 0.18 \quad (\text{average})$$

Alternatively, from Fig. 11, for $r_{u,max} = 0.77$ and $\gamma = 0.1\%$, r_u is estimated to be 0.4 .

In summary, the partially saturated sand layer in the example presented with $S = 80\%$ and $D_r = 30\%$, during an earthquake with $M = 7$ causing a maximum (peak) shear strain of 0.17% , will not liquefy, but it may experience an excess pore pressure ratio of up to $r_u = 0.4$. Excess pore pressures generated in partially saturated sands

can be of importance in geotechnical earthquake engineering in the estimation of soil strength and settlement.

Summary and Conclusions

Partially saturated sands can be encountered in the field occurring naturally or induced through the introduction of gas bubbles as a measure for liquefaction mitigation. To evaluate the liquefaction response of partially saturated sands, laboratory tests were performed using ranges of the degree of saturation, relative density, and cyclic shear strain amplitude. The results of the tests were interpreted in terms of excess pore pressure ratios. The experimental data for $S < 96.3\%$ indicate that partially saturated sands do not liquefy but can develop excess pore pressures depending on the degree of saturation, relative density, and amplitude and number of shear strain cycles. For the prediction of r_u in partially saturated sands experiencing seismic excitation, an empirical model (RuPSS) was developed utilizing laboratory data. The model was developed in two stages. In the first stage, a function was established that relates the maximum excess pore pressure ratio ($r_{u,max}$) to the degree of saturation, relative density, and cyclic shear strain amplitude. In the second stage, excess pore pressure ratio (r_u) generation was expressed in terms of the maximum excess pore pressure ratio ($r_{u,max}$), the number of cycles required to achieve this maximum, and the number of equivalent shear strain cycles associated with an earthquake-induced shear strain time history. Combining the functions developed from these two stages, the empirical model RuPSS for the prediction of r_u in partially saturated sands was established.

With RuPSS, plots were prepared for easy and conservative estimation of the excess pore pressure ratio (r_u) in partially saturated sands subjected to seismic excitation. To illustrate the various steps of the procedure involved in the application of RuPSS, an example soil profile was evaluated.

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