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RELIABILITY OF LATERALLY LOADED PILE ANALYSIS

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ABSTRACT

This paper describes a procedure for estimating the uncertainties in the computed deflections and bending moments in a pile subjected to lateral loads. In this procedure, the uncertainties in the soil properties and soil response are first estimated; then, employing the finite difference equations governing the pile behavior, estimates are made of the uncertainties in the computed curves which describe the lateral behavior of the pile.

Several example studies are presented in which a thirty-inch diameter pile embedded in clay is analyzed to obtain the uncertainties in the computed pile behavior as a function of the uncertainties in the input parameters. The results show that the accuracy of the solutions to the problem of a laterally loaded deep foundation obtained using the finite difference technique is very much dependent upon the accuracy of the predicted average soil properties and soil response.

Of all the sources of uncertainties studied in this paper, the uncertainties in the undrained strength of the clay and in the ultimate resistance coefficient, which together define the ultimate lateral soil resistance, contribute most significantly to the uncertainties in the computed pile deflections and bending moments.

INTRODUCTION

The design of a laterally loaded pile involves the estimation of the pile deflections and bending moments corresponding to some design load (5). Insofar as uncertainties exist in the analysis procedure and in the input parameters, the computed pile behavior will also be inexact. Therefore, the actual behavior of the pile may not necessarily be the same as that predicted by the analysis. To account for the possible variation in pile behavior and to safeguard against failure of the pile, the common practice is to use conservative estimates of the input data and then to adopt a factor of safety in the design, thus "ensuring" that in the future the pile deflections and bending moments will be within design specifications. Often, such a practice is overconservative and results in unnecessarily expensive designs.

An alternate approach to the design of a laterally loaded pile is to first estimate the "expected" behavior of the pile, based on the average values of the input data, and then to obtain estimates of the uncertainties in the computed results. This approach enables the design engineer to assess the risk of failure for different design alternatives. The "best" design will then correspond to that which satisfies both economic considerations and a particular level of acceptable risk of failure (10).

The purpose of this paper is to present a procedure for estimating the uncertainties in the computed deflections and bending moments of a pile subjected to lateral loads and to study the reliability of the analysis for laterally loaded piles using the procedure developed. This procedure is based upon a probabilistic model which employs the uncertainties in the soil properties and soil response together with the finite difference equations for the solution of the pile problem. In the present paper, only piles in clays are considered.

LATERALLY LOADED PILE ANALYSIS

The analysis of a laterally loaded pile involves the solution of the differential equation which governs the lateral behavior of the pile. Commonly, the finite difference technique is employed to obtain pile deflections and bending moments compatible with the lateral displacements of the soil surrounding the pile (7). In this procedure, the soil and its resistance to lateral movements at different depths are described by a set of "p-y" curves as shown in Fig. 1. These curves are developed employing empirical procedures based upon soil properties and pile geometry (3,4). Inherent in this procedure is the assumption that the behavior of the soil at a particular depth is independent of the soil behavior at all other depths.

Most criteria currently employed to describe the soil resistance-displacement relationships for clays

References and illustrations at end of paper.

can be expressed as

$$p = 0.5\, p_u \left(\frac{y}{y_{50}}\right)^m \text{ .... (1)}$$

where p_u is the ultimate soil resistance, y is the pile deflection, y_{50} is the pile deflection at one half of p_u, and m is a parameter which defines the shape of the p-y curve. Fig. 2 shows a typical p-y curve.

The deflection at one-half the ultimate soil resistance can be obtained using Skempton's (1951) expression which relates y_{50} to the soil strain, ε_{50} at one-half the maximum principal stress difference and D, the pile diameter.

$$y_{50} = 2.5\, D\, \varepsilon_{50} \text{ .... (2)}$$

The ultimate soil resistance, p_u, is commonly expressed as:

$$p_u = N_p \cdot S_u \cdot D \text{ .... (3)}$$

where S_u is the undrained shear strength of the soil, and N_p is referred to as the ultimate resistance coefficient.

Various investigators in the past have studied the ultimate lateral resistance of clays and have proposed procedures to calculate N_p (2,3,4,6). The most widely used criterion for estimating N_p is that proposed by Matlock (1970). In this criterion, the value of N_p is empirically determined depending upon the relative depth below the ground surface.

Yegian and Wright (1973) studied the ultimate lateral resistance of piles using the finite element method. Special interface elements were employed in the analysis to allow relative displacements between the soil and the pile. The ultimate soil resistances predicted by the finite element analyses disagreed with those found using Matlock's criterion by as much as 33%, depending on the soil-pile interface properties used.

Inasmuch as the criteria employed to obtain p, p_u, and y_{50} are basically empirical and uncertainties in the soil parameters used in these criteria always exist, the p-y curves developed using these empirical approaches will also be uncertain.

SOURCES OF UNCERTAINTIES

Among the sources of uncertainties involved in the analysis for laterally loaded piles employing the finite difference technique are:

1. The nonlinear soil resistance-displacement relationships describing the lateral behavior of the soil;
2. The load on the pile and the boundary conditions;
3. The accuracy of the solution of the differential equation using the finite difference technique;
4. The pile rigidity and pile shape which might be different from the expected shape because of probable faulty construction.

This paper is limited in scope to the study of the influence of the first source of uncertainty upon the computed motions. Therefore, the estimates of uncertainty in computed pile behavior presented in this paper reflect only the uncertainty in the model describing the soil behavior around the pile.

PROBABILISTIC MODEL

For the sake of mathematical convenience, the lateral deflection of a pile segment is expressed in the finite difference formulation as:

$$y_i = a_i + b_i y_{i+1} \text{ .... (4)}$$

where a_i and b_i are coefficients which depend on the pile rigidity and the soil modulus corresponding to the pile segment i. The computed deflection at the top of the pile y_t is therefore a function of all the a and b coefficients along the pile and the boundary conditions at the top of the pile,

$$y_t = f_1\, (a_i, b_i, \quad i=1,N \text{ \& boundary conditions}) \quad (5)$$

where N is the total number of pile segments plus one. Considering only the influence of the uncertainty in the soil modulus, the variance of y_t will be a function of the variances of the soil moduli along the pile,

$$\mathrm{Var}[y_t] = f_2\, (\mathrm{Var}[E_{si}], \quad i=1, N) \text{ .... (6)}$$

where E_s is the soil modulus estimated from the average nonlinear p-y curve. Since the p-y curves are uncorrelated with depth, $\mathrm{Var}[y_t]$ can be approximated as (1)

$$\mathrm{Var}[y_t] = \sum_{i=1}^{N} \left(\frac{\partial y_t}{\partial E_{si}}\right)^2_{\text{at } \overline{E}_{si}} \cdot \mathrm{Var}[E_{si}] \text{ .... (7)}$$

where

$$\frac{\partial y_t}{\partial E_{si}} = f_3\left(\frac{\partial a_i}{\partial E_{si}}, \frac{\partial b_i}{\partial E_{si}}, \quad i=1,N\right) \text{ .... (8)}$$

and is evaluated using average values of the soil moduli.

Having thus estimated $\mathrm{Var}[y_t]$, the variance of the deflection, y, at any other point along the pile can be estimated by repeated applications of Eqs. 4 and 7.

The variance of the computed average bending moment can also be approximated by

$$\mathrm{Var}[M] = \sum_{i=1}^{N} \left(\frac{\partial M}{\partial E_{si}}\right)^2_{\text{at } \overline{E}_{si}} \cdot \mathrm{Var}[E_{si}] \text{ .... (9)}$$

where $\frac{\partial M}{\partial E_{si}}$ is obtained from the finite difference expression for the moment.

The computer program, COM622, developed by Reese (7) which provides deterministic solutions for a laterally loaded pile using the finite difference technique was modified so that it would produce estimates of the uncertainties in the computed pile behavior.

These uncertainty estimates, obtained following the procedure described, are in terms of the variances or the coefficients of variation of the pile deflections and bending moments. These estimates could be used together with a probability density function (PDF) for the deflections or the moments to make predictions of the expected pile behavior corresponding to some "confidence" level. Very often, these probability density functions are not known and are extremely difficult to obtain in most studies. For the purposes of convenience, the probability density function of the moments is assumed in this analysis. Benjamin and Cornell (1970) discuss the lognormal distribution as best to represent a phenomenon arising from multiplicative mechanisms. Since the finite difference formulation for the solution of a laterally loaded pile involves such a process, the lognormal distribution may be a reasonable assumption for the PDF of the moments. It is emphasized herein that this assumption is made for the sake of convenience only.

VARIANCE OF SOIL MODULUS

To obtain the variances of the pile deflections and bending moments along the length of the pile, the variances of the soil moduli with depth are required. In a previous section it was discussed that the p-y curves are employed to relate the soil resistance at a point to the pile deflection at the same point. The computed pile deflections with depth are based on average soil moduli which yield soil resistances and deflections along the pile compatible with the average p-y curves as shown in Fig. 2. The soil modulus at a particular depth can be expressed as,

$$E_s = \frac{p}{y} \quad \text{....................................(10)}$$

Combining the expressions for p, p_u and y_{50}, Eq. 10 can be rewritten as:

$$E_s = \frac{0.5\, N_p S_u D (y)^{m-1}}{(2.5\, D\ \varepsilon_{50})^m} \quad \text{.....................(11)}$$

The variance of E_s can be approximated by (1):

$$Var[E_s] = \sum_{i=1}^{N} \left(\frac{\partial E_s}{\partial X_i}\right)^2_{\bar{X}_i} Var[X_i] \quad \text{............(12)}$$

where X_i is a variable defining E_s. Substituting Eq. 11 into Eq. 12:

$$Var[E_s] = \bar{E}_s^2 \left[\frac{Var[S_u]}{\bar{S}_u^2} + \frac{m^2 Var[\varepsilon_{50}]}{\bar{\varepsilon}_{50}^2} + \frac{Var[N_p]}{\bar{N}_p^2} + Ln^2\left(\frac{y}{y_{50}}\right) \cdot Var[m] + \frac{(m-1)^2\, Var[y]}{\bar{y}^2} \right] \quad (13)$$

and the coefficient of variation of E_s is given by:

$$V_{E_s} = \sqrt{\frac{Var[E_s]}{\bar{E}_s^2}} \quad \text{..........................(14)}$$

The first and second terms on the right hand side of Eq. 13 are the contributions of the uncertainties in the soil properties to the uncertainty in the soil modulus. The third term can be considered to be a measure of both the uncertainty in the ultimate resistance coefficient and the uncertainty in the criterion used in relating p to y. The fourth term in Eq. 13 is an estimate of the uncertainty in the shape or form of the p-y curve. The last term is the uncertainty in the soil modulus because of uncertainty in the computed pile deflection.

Eq. 7 expresses Var $[y_t]$ as a function of Var $[E_s]$ which, in turn, is a function of Var $[y_t]$ as shown in Eq. 13. It is therefore clear that an iterative procedure is required in order to obtain Var $[y_t]$ from Eq. 7, consistent with Var $[E_s]$ obtained from Eq. 13. In most cases Var $[y_t]$ is small relative to the variances of the other parameters shown in Eq. 13. Ordinarily, convergence is achieved in one or two iterations.

EXAMPLE STUDY

Reese and Welch (1975) presented the results of a load test on a thirty-inch diameter pile subjected to various lateral loads. Employing these load test results, a criterion was developed and proposed for p-y curves for stiff clays. Using the computer program COM622 and the criterion proposed, comparisons were made by the investigators between the computed bending moments in the pile and the results from the load tests. In most cases the computed average bending moments agreed well with the measured data except when the lateral load on the pile was 30 tons. In view of the fact that significant uncertainties were present in the soil properties used and in the criterion proposed for generating the p-y curves for the soil, this disagreement is not very surprising.

In order to assess the influence of these uncertainties upon the computed bending moments, a reliability analysis was performed on the same 30-inch diameter pile employing the modified version of the computer program COM622. The average soil properties, the p-y curves, and the section moduli of the pile were the same as those used by the original investigators. The coefficients of variation of S_u and m were estimated from the available data to be equal to 0.33 and 0.07 respectively. Also, the coefficient of variation of ε_{50} and N_p were assumed to be equal to 0.50 and 0.25 respectively. Fig. 3 shows the results of the reliability study. The average curves shown in Fig. 3 agree very well with those predicted by Reese and Welch. From this figure it is clear that, employing average soil properties, the bending moments as predicted from COM622 are slightly overestimated. Accounting for the uncertainties both in the soil properties and in the p-y criterion, estimates were made of the variances of the computed bending moments with depth.

Employing these estimates and the lognormal distribution, upper and lower bounds for the bending moments were established. Fig. 3 shows the average computed bending moments with depth together with the established bounds. Associated with this band for the moments is a 98% confidence level that the actual moments in the field will not be greater than or less than the values defined by the band. Fig. 3 also shows values of the bending moments measured in the field. Since most of the measured data plot within the band defined, it may be stated that the reliability study performed and the uncertainty estimates made appear to be reasonable.

The example study, presented herein, suggests that, in order to make reasonable predictions of pile deflections and bending moments, the uncertainties in the computed average values should be estimated and the predictions be made corresponding to some level of "confidence."

ADDITIONAL STUDIES

The expression shown in Eq. 13 indicates five parameters contributing to the uncertainties in the soil moduli used to compute pile deflections and bending moments. It is worthwhile herein to assess the relative influences of each of these five parameters upon the computed pile behavior. To achieve this purpose the pile discussed in the previous example was chosen for study and various reliability analyses were performed by changing the lateral load on the pile, the strength of the clay and the uncertainties in the soil strength and the soil response.

To establish the maximum and minimum influences of the uncertainties in E_s upon the computed deflections and bending moments, four combinations of loads and soil types were employed as follows:

a) $P_T = 30^T$, stiff clay, $S_u = 2200$ psf, and $m = \frac{1}{4}$

b) $P_T = 60^T$, stiff clay, $S_u = 2200$ psf, and $m = \frac{1}{4}$

c) $P_T = 30^T$, soft clay, $S_u = 500$ psf, and $m = \frac{1}{3}$

d) $P_T = 50^T$, soft clay, $S_u = 500$ psf, and $m = \frac{1}{3}$.

Loading conditions a and b were chosen to study the influence of Var $[E_s]$ when the soil moduli are large and the deflections are small. Conditions c and d correspond to small soil moduli and very large deflections.

To determine the uncertainties in the computed results as a function of the uncertainties in the input parameters, for each of the loading conditions five cases were studied. These five cases corresponded to different levels of uncertainty in the undrained shear strength of the clay. Table 1 summarizes the data for each of the five cases. For all cases, the coefficients of variation,

$$\left(\frac{\text{standard deviation}}{\text{mean}}\right)$$

of N_p, ε_{50}, and m were assumed to be constant and equal to 0.25, 0.50, and 0.07 respectively. In all the analyses the length of the pile was 42 feet, $EI = 2.8 \times 10^{12}$ psi, and the pile was considered to be a free ended pile with no axial force or moment at the top.

The relative influences of each of the uncertain parameters upon the estimated soil modulus E_s are shown in Table 1. Column 1 shows the coefficient of variation of E_s, if only S_u is uncertain and all other parameters are deterministic. Similarly, Column 2 shows V_{E_s} due to uncertainty only in N_p. Column 3 shows V_{E_s} due to uncertainties in both S_u and N_p considered together.

The variances of S_u, N_p, and ε_{50} were assumed be constant with depth. Therefore, for each case studied, V_{E_s} due to these parameters is constant with depth and unique. However, an inspection of Eq. 15 will show that V_{E_s} due to the five parameters, i.e., S_u, N_p, ε_{50}, y, and m will vary both with depth and with the computed deflections. Therefore, for each case described in Table 1, considering uncertainties in all the five parameters, there is a range of values of V_{E_s} corresponding to different depths and loading conditions. Column 4 shows the range of V_{E_s} for each case studied.

Comparing the values of V_{E_s} from Column 3 with those from Column 4, it can be stated that, considering a wide range of loading conditions, the total uncertainty in the soil modulus is not significantly different from the uncertainty in E_s due to only S_u and N_p. In other words, if the coefficient of variation of E_s due to S_u and N_p is greater than 0.20, uncertainties in E_s due to uncertainties in ε_{50}, y, and the form or shape of the p-y curve are relatively unimportant.

Insofar as uncertainties in S_u will always be present, the coefficient of variation of E_s can therefore be approximated by:

$$V_{E_s} = \sqrt{\frac{\text{Var}[S_u]}{\bar{S}_u^{\,2}} + \frac{\text{Var}[N_p]}{\bar{N}_p^{\,2}}} \quad \text{.............(15)}$$

The results of the reliability analyses are shown in Fig. 4. The horizontal axis is the coefficient of variation of the soil modulus at the top of the pile. The vertical axis describes the uncertainty in the computed average pile deflections in terms of its coefficient of variation.

Similarly, Fig. 5 shows the coefficient of variation of the computed maximum bending moment as a function of the coefficient of variation of E_s.

From Figs. 4 and 5 it can be observed that for a given coefficient of variation of E_s, the softer the soil or the larger the deflections, the smaller the coefficient of variation of both the top deflection and the maximum bending moment. This can be attributed to the nonlinear characteristics of the p-y curves. For a given coefficient of variation of E_s, the larger the pile deflections the smaller the soil moduli and consequently the smaller the variance of E_s. It should be noted herein that even though the coefficients of variation of the top deflection and the maximum bending moment are smaller for the soft soil than the stiff soil, the actual variances of the deflection and the maximum bending moment for the soft soil could be significantly larger than if the soil were stiff. In view of the fact that the loading conditions corresponding to 30 tons and stiff clay and 50 tons and soft clay were chosen to describe the widest range of uncertainties in the deflections and moments, the two lines in Figs. 4 and 5 roughly define this range.

The results shown in Figs. 4 and 5 indicate that for $V_{E_s} = 0.60$, the coefficients of variation of the top deflection and maximum bending moment of the 30-inch diameter pile studied can be as large as 16% and 13% respectively. Using the lognormal distribution and the coefficients of variation of the top deflection (16%) and the maximum moment (13%), upper and lower estimates were made of the pile behavior with 98% confidence that the actual field values of the top deflection and maximum bending moment will be within the estimated ranges.

The results show that corresponding to the 98% confidence interval, the top pile deflection will range between 0.68 to 1.45 x the average computed value. The corresponding confidence interval for the bending moment ranges from 0.73 to 1.35 x the average computed value. In other words, for the example pile studied in which the coefficient of variation of the soil modulus is 0.60, the top pile deflection and maximum bending moment computed using average soil parameters might be underestimated by as much as 45% and 35% respectively or overestimated 32% and 27% respectively. These uncertainty estimates are, of course, valid only for the example pile studied and can be exceeded in other pile studies, depending upon the coefficient of variation of E_s.

Finally, it is emphasized herein that the reliability analysis for laterally loaded piles presented in this paper considers only the uncertainties in the soil model. Therefore, considering also the other sources of uncertainties discussed in this paper, the total expected error in the computed pile behavior would be significantly greater than the value obtained following the procedure presented.

CONCLUSIONS

It is essential that in the design of a laterally loaded pile, considerations be given to the uncertainties present in the soil parameters used and the model employed in the analysis to describe the lateral behavior of the soil surrounding the pile. This paper describes a procedure for estimating the uncertainties in the computed average deflections and bending moments in a pile subjected to lateral loads. The method is based on a probabilistic model which employs the finite difference equations governing the behavior of the pile. The computer program COM622 has been modified in order to provide the variances in the computed deflections and bending moments in addition to the average pile curves. This modified computer program appears to yield reasonable estimates of the uncertainties in the computed pile behavior as a function of the uncertainties in the input soil parameters.

Preliminary analyses on an example pile using the reliability model presented indicate that the accuracy of the computed average pile deflections and bending moments largely depend on the degree to which the soil strength and soil response can be predicted.

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TABLE 1: Coefficients of Variation of Soil Modulus, V_{E_s}

			V_{E_s}	
CASE	$\sqrt{\frac{Var[S_u]}{\bar{S}_u^2}}$	$\sqrt{\frac{Var[N_p]}{\bar{N}_p^2}}$	$\sqrt{\frac{Var[S_u]}{\bar{S}_u^2}+\frac{Var[N_p]}{\bar{N}_p^2}}$	FROM EQ. 14
1	0	0	0	0.10 - 0.18
2	0	0.25	0.25	0.27 - 0.30
3	0.20	0.25	0.32	0.32 - 0.35
4	0.33	0.25	0.42	0.45 - 0.48
5	0.56	0.25	0.62	0.63 - 0.67

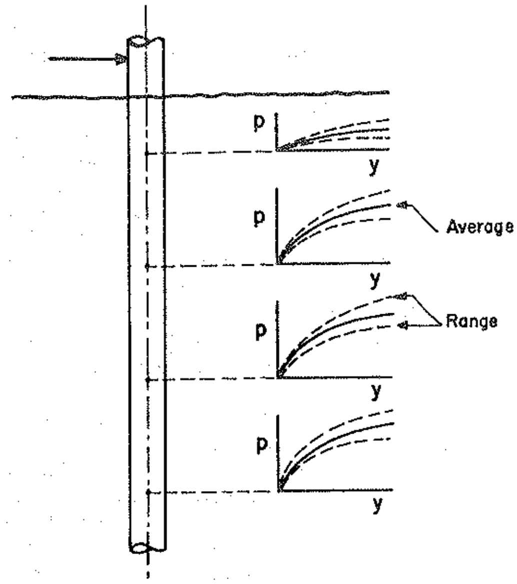

Fig. 1 - Typical laterally loaded pile.

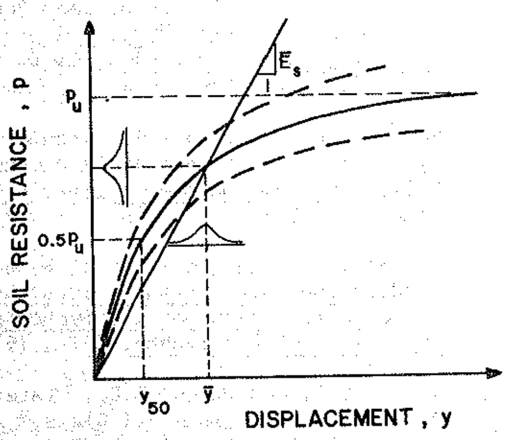

Fig. 2 - Typical p-y relationship and average soil modulus.

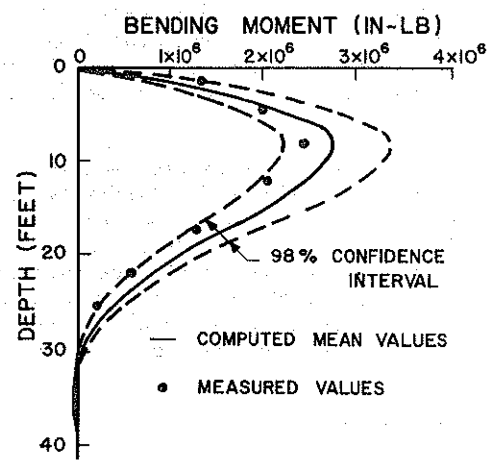

Fig. 3 - Measured and computed values of bending moment with depth.

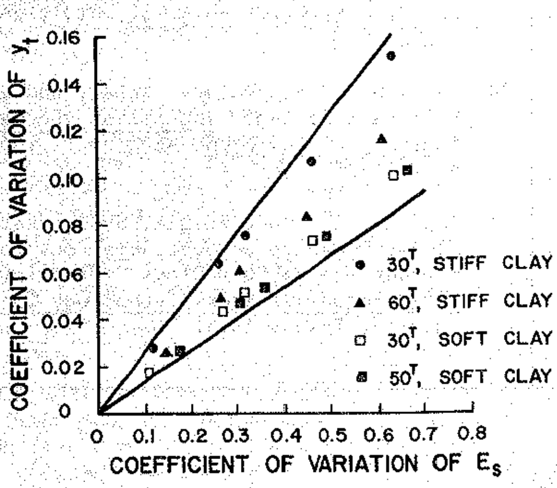

Fig. 4 - Coefficient of variation of top deflection, y_t versus coefficient of variation of soil modulus, E_s.

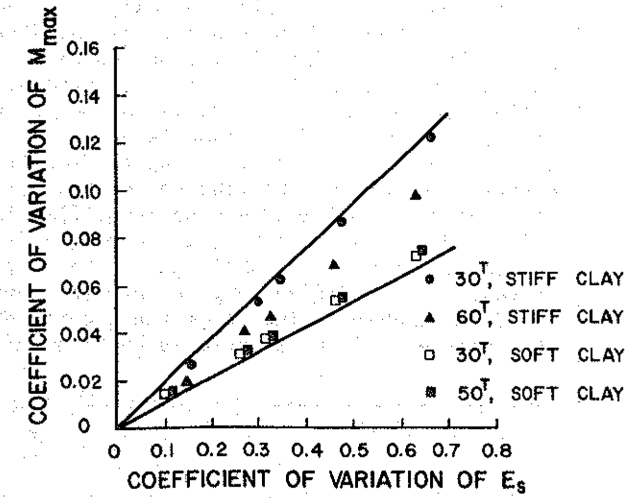

Fig. 5 - Coefficient of variation of maximum bending moment, M_{max} versus coefficient of variation of soil modulus, E_s.