Seismic Risk Analysis for Earth Dams

By M. K. Yegian,[1] E. A. Marciano,[2] Members, ASCE, and V. G. Ghahraman,[3] Associate Member, ASCE

Abstract: There are many sources of uncertainty involved in seismic safety evaluation of an embankment or an earth dam. Even when conservative assumptions and selections of parameters are made, there will always be a probability that performance of the dam during its lifetime may not be as predicted. To evaluate seismic risk of damage or failure of earth dams, a seismic risk analysis procedure is presented. The future occurrence of earthquakes is probabilistically described in terms of both intensity and number of cycles of ground motion (seismic hazard analysis). Furthermore, a probabilistic procedure for the calculation of permanent deformation of earth dams (seismic performance analysis) is presented, characterizing the seismic event in terms of acceleration, number of cycles, and predominant period of motion. The results of seismic hazard and seismic performance analyses are combined to yield the risk of seismic damage or failure of a dam (seismic risk analysis). An example case study is presented, illustrating the application of the procedures. A risk-based seismic safety evaluation of an embankment or an earth dam can provide estimates of relative risks that are useful in design and decision analysis and can avoid compounding of conservatism.

Introduction

Over the past two decades, significant developments have been made in understanding the dynamic response of slopes and earth dams. Analytical procedures have been developed to varying degrees of sophistication ranging from the simple application of Newton's second law to three-dimensional finite element analyses. Despite this progress, estimation of the likelihood of seismically induced failure, or conversely, of the reliable performance, of an earth dam during its functional life remains a challenge.

Seismic safety evaluation of an earth dam involves the identification and determination of various parameters that define the seismic loads on the dam and the resistance of the dam to these loads. For a realistic evaluation of the seismic risk, one must consider many uncertainties such as those related to the seismicity and geology of the site, the stiffness and strength of the foundation and dam materials, and even the methods of analyzing the response of the dam.

The current practice of seismic safety evaluation of an earth dam favors a deterministic approach. To account for the various uncertainties, conservative selections of parameters and assumptions are made. Typically, a minimum required factor of safety or a limiting level of permanent strain or deformation is adopted to ensure safety. This approach can render the design of a new earth dam economically unfeasible because of the potential of compounding conservatism. Furthermore, in the seismic safety evaluation of an

[1]Prof., Dept. of Civ. Engrg., 420 Snell Engrg. Ctr., Northeastern Univ., Boston, MA 02115.

[2]Asst. Prof., Dept. of Civ. Engrg., Northeastern Univ., Boston, MA.

[3]Grad. Student, Dept. of Civ. Engrg., Northeastern Univ., Boston, MA.

Note. Discussion open until June 1, 1991. Separate discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on March 19, 1990. This paper is part of the *Journal of Geotechnical Engineering*, Vol. 117, No. 1, January, 1991. ©ASCE, ISSN 0733-9410/91/0001-0018/$1.00 + $.15 per page. Paper No. 25413.

existing earth dam, the deterministic approach can lead to the conclusion that the dam is unsafe, whereas a more realistic evaluation accounting for the uncertainties in the parameters, assumptions, and methods of analysis might indicate that the level of risk is acceptable to all parties concerned.

This paper describes a probabilistic approach to the seismic safety evaluation of earth dams. The method involves the integration of seismological and geotechnical inputs and their uncertainties in a consistent manner to yield the likelihood of seismically induced damage and catastrophic failure of a dam. The application of this approach can be used to compare the effects of alternative design options on the seismic risk and the cost of construction. It can also help to identify the most important parameters, assumptions, and design criteria affecting the evaluation of seismic safety. Most importantly, it provides a means by which a designer can avoid the trap of compounded conservatism, which is typical of the deterministic approach (Peck 1977).

Methodology

Seismic risk analysis is performed in the following three steps.

1. Seismic hazard analysis (SHA) accounts for the various seismic sources and the characteristics of the seismic excitations that can be generated by these sources. It yields probabilistic statements of the recurrence of different levels of

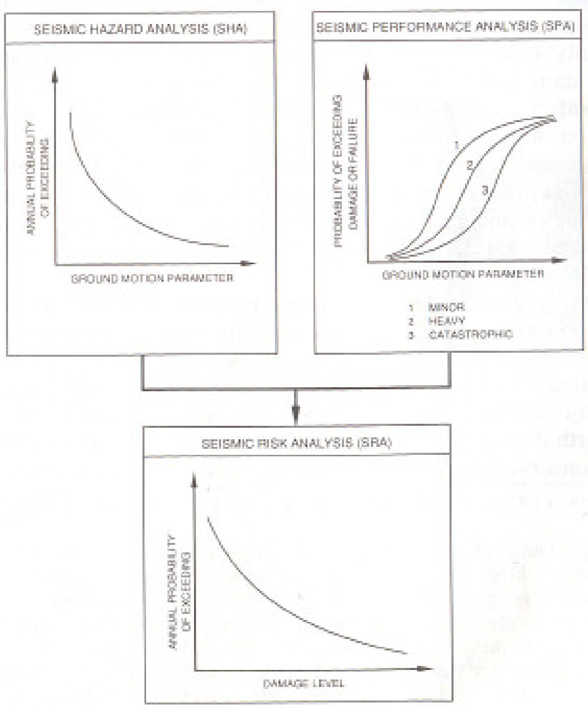

FIG. 1. Steps Involved in Seismic Risk Analysis

seismic hazard defined in terms of ground motion parameters at the site of a facility.

2. Seismic performance analysis (SPA) accounts for the seismic resistance of the facility. It provides probabilistic statements regarding the performance of the facility, conditioned upon specified levels of seismic excitation.

3. Seismic risk analysis (SRA) integrates the results of SHA and SPA to yield the overall risk of damage or failure. Fig. 1 is a schematic representation of the integration involved in SRA.

The methodology described herein is applicable to any type of facility (Whitman 1984). Procedures were developed to apply this methodology specifically to earth dams. A matrix approach is used to display the results of the probability calculations involved in the three steps of the risk analysis. There are advantages to using the matrix approach instead of closed-form formulations. The integration of the results from SHA and SPA to determine the seismic risk can be done with a hand-held calculator. This makes it convenient to repeat analyses to assess the effects of different assumptions and different design alternatives upon seismic risk. Furthermore, the matrix approach permits the use of simple, or, if deemed necessary, more rigorous state-of-the-art analyses to obtain the damage probabilities. For example, SPA can be performed empirically, subjectively, using simple analytical procedures, or using three-dimensional finite element analyses. The following sections describe the procedures involved in the calculation of seismic risk analysis for earth dams.

Modes of Damage

An earth dam can incur seismic damage by different mechanisms, often referred to as modes of failure. Two modes of seismic damage are considered herein. Mode 1 is permanent deformations of the dam that accumulate during the course of the earthquake and could lead to overtopping of the dam due to loss of freeboard. Mode 2 is the development of instability at the end of the earthquake due to reduction of the shear strength of the foundation and dam materials during the shaking.

Mode 1 was first considered analytically by Newmark (1965) using a simple rigid-plastic force-displacement relationship. In recent years, other analytical procedures (Ambraseys and Menu 1988; Chaney 1979; Franklin and Chang 1977; Gazetas 1987; Lin and Whitman 1986; Makdisi and Seed 1978; Taniguchi et al. 1983) for calculating permanent deformations have been developed. Many of these procedures are essentially extensions of Newmark's simple concept. A review of these and other methodologies indicates that the characteristics of seismic excitation play an important role in the estimation of permanent deformation of an earth dam. The peak ground acceleration, frequency content, duration, and random nature of the ground motions affect the magnitude of the accumulated permanent deformation.

The likelihood of mode 2 (namely, of a postearthquake stability failure) depends not only on the severity of seismic ground motions, but also on the postearthquake characteristics of the foundation soil and dam materials.

In the companion paper (Yegian et al. 1991), a simple procedure for calculating permanent deformations of an earth dam is presented. The parameters selected to describe the seismic excitation are peak ground acceleration, predominant period of motion, and number of equivalent cycles of seismic load application, N_{eq}. The selection of number of equivalent cycles as one of the parameters is consistent with the procedures for postearthquake instability analysis involving the cyclic shear strength of soils.

Seismic Hazard Analysis (SHA)

For a specific level of peak ground acceleration, the probability of exceeding a specified permanent deformation and the probability of postearthquake instability of a dam will be lower for a smaller number of cycles of motion. Therefore, the SHA procedure for an earth dam must determine, in addition to the annual number of events causing acceleration to exceed a specified level, the distribution of these events with respect to the number of cycles of excitation.

Current SHA procedures typically provide the number of events causing acceleration to exceed a specified level (Cornell 1968; Idriss 1985; Reiter and Jackson 1983; Schumacker and Whitman 1978; Yegian 1979). The results are expressed in terms of annual number of events causing acceleration A to exceed a specified value a, $\lambda(A \geq a)$. However, this total number of events $\lambda(A \geq a)$ will generally have contributions from different ranges of seismic magnitudes, M, due to varying site-to-source distances associated with different seismic sources.

The mathematical formulation that permits the calculation of the distribution of $\lambda(a, \Delta M)$, where ΔM is the range of earthquake magnitude, is explained in detail by Yegian et al. (1988). The number of earthquakes causing acceleration at a site to exceed a and having magnitudes between the minimum value of interest, m_0 and a selected value, m_i can be obtained through the use of computer programs currently used in conventional probabilistic SHA, provided that the following input parameter modifications are made.

1. The maximum credible magnitude to be read into the computer should be m_i, instead of m_{max}.
2. In lieu of the number of events, $\lambda(m_0)$ having magnitudes between m_0 and m_{max}, input the number of events exceeding m_0 but less than m_i, which equals

$$\lambda(m_0)\,\frac{1 - e^{-\beta(m_i - m_0)}}{1 - e^{-\beta(m_{max} - m_0)}} \quad (1)$$

where β = the magnitude-frequency parameter defined by Cornell (1968).

Using these modified parameters, the results of the computer analysis will correspond to $\lambda(a, \Delta M_i)$, the total number of events causing acceleration to exceed a and having magnitudes greater than m_0 but less than m_i. By selecting various values of m_i, it is possible to generate a histogram of events $\lambda(a, \Delta M)$ for various intervals of magnitude.

To establish the joint occurrence of acceleration and number of cycles, $\lambda(a, \Delta N_{eq})$, a relationship between Richter magnitude, M, and number of equivalent cycles, N_{eq}, is required. Yegian et al. (1988) present a summary of various relationships proposed in the literature by different investigators. Using any one of these relationships, the distribution of $\lambda(a, \Delta N_{eq})$ can be determined from the distribution of $\lambda(a, \Delta M)$.

Example Application of SHA

An example case study was performed for the embankment profile illustrated in Fig. 2. The SHA portion of the case study, which provides the joint distributions of $\lambda(a, \Delta M)$ and $\lambda(a, \Delta N_{eq})$, is discussed in this section. The seismic sources and parameters used in this analysis were obtained from Cornell and Merz (1974) and Tong et al. (1975). The computer program developed by Schumacker and Whitman (1978) was used for the SHA. Using the SHA procedures described in the previous section, the expected annual number of events, $\lambda(\Delta A, \Delta M)$, having acceleration and Richter magnitude within the ranges ΔA and ΔM, respectively, were computed. The results are presented in matrix form in Table 1. The Richter magnitude values were converted to the number of equivalent cycles, N_{eq}, using the relationship proposed by Seed et al. (1983), thus giving $\lambda(\Delta A, \Delta N_{eq})$, which is also shown in Table 1.

Fig. 3 shows the resulting histogram of the number of events as a function of M and N_{eq} for peak accelerations exceeding 0.15g. It can be seen that for this example the magnitude, M, and the number of cycles, N_{eq}, associated with an acceleration of 0.15g will most likely be small. This information is extremely useful in the analysis of permanent deformations and evaluation

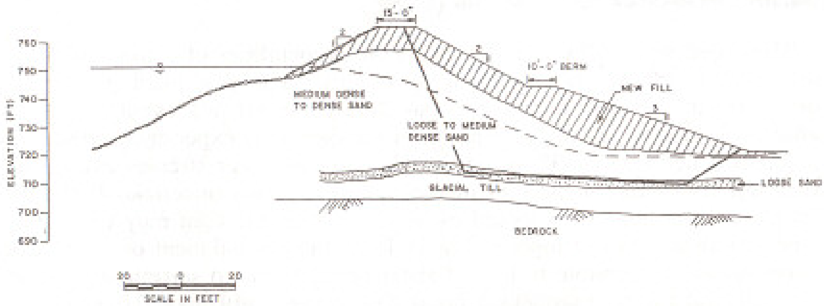

FIG. 2. Cross Section of Embankment Considered in Example Case Study (1 ft = 0.305 m)

TABLE 1. Seismic Hazard Analysis Results for Example Site

Magnitude (1)	Number of cycles (2)	ANNUAL NUMBER OF EVENTS, λ (10^{-4}) — Peak Ground Acceleration (g): $0.0 \leq A < 0.05$ (3)	$0.05 \leq A < 0.1$ (4)	$0.1 \leq A < 0.15$ (5)	$0.15 \leq A < 0.2$ (6)	$0.2 \leq A < 0.25$ (7)	$A \geq 0.25$ (8)
$4.33 \leq M < 5.0$	$1 \leq N_{eq} < 2$	1,467.50	41.22	7.87	2.67	1.07	1.25
$5.0 \leq M < 5.5$	$2 \leq N_{eq} < 3$	392.80	18.51	4.17	1.25	0.55	0.64
$5.5 \leq M < 6.0$	$3 \leq N_{eq} < 5$	163.30	12.04	2.54	0.98	0.37	0.46
$6.0 \leq M < 6.5$	$5 \leq N_{eq} < 8$	68.19	7.22	1.86	0.73	0.36	0.44
$6.5 \leq M < 6.8$	$8 \leq N_{eq} < 11$	12.90	1.72	0.67	0.21	0.12	0.18
Total	—	2,113.69	80.71	17.11	5.84	2.47	2.97

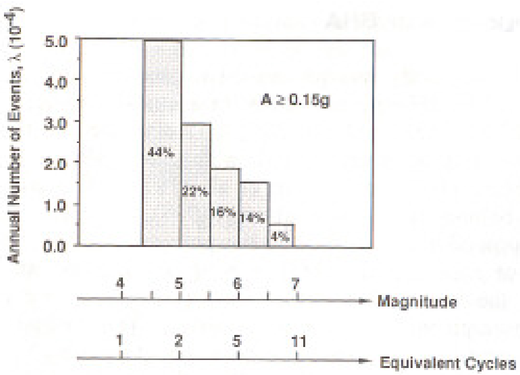

FIG. 3. Histogram of Annual Number of Events

of postearthquake shearing resistance. Similar histograms were generated for peak accelerations exceeding 0.0g, 0.05g, 0.1g, 0.2g, and 0.25g. These were used to generate Table 1.

Seismic Performance Analysis (SPA)

The objective of SPA is to determine the probabilities of damage and failure of an earth dam conditioned upon a specified seismic loading. Damage or failure of an earth dam can occur during the seismic event or shortly afterward. During shaking, a section of the dam may experience substantial permanent deformation if the earthquake-induced shear stresses exceed the shearing resistance of the embankment or foundation materials. If the deformation reduces the freeboard excessively, then the dam may experience total failure due to overtopping (mode 1). If the embankment or foundation materials are susceptible to loss of shear strength due to seismic excitation, the potential for postearthquake instability of the dam (mode 2) exists.

In the companion paper, a procedure is described for calculating the probabilities of the permanent deformation exceeding different levels (mode 1) conditioned upon the occurrence of specified seismic events. The results of this procedure can be presented in matrix form as shown in Table 2. Table 2 shows the mode 1 results of the SPA performed for the embankment cross section shown in Fig. 2, which is discussed in detail in the latter part of this paper.

The evaluation of the postearthquake stability of an earth dam (mode 2) is typically performed by conventional slope stability analysis using the shear strengths corresponding to the end of the earthquake (Hadj-Hamou and Kavazanjian 1984; Ishihara 1984; Mohamad et al. 1985; Ramanujam et al. 1978; Seed et al. 1975; Seed 1987). During the past decade, empirical, experimental, and analytical procedures for estimating postearthquake shear strengths and pore water pressures have been developed (Bergado and Anderson 1985; Castro 1976; Dobry et al. 1982; Kikusawa and Hasegawa 1985; Klohn et al. 1978; Ramanujam et al. 1978; Seed 1987; Yegian and Vitelli 1981; Yegian 1984). In addition, probabilistic procedures have been formulated to account for uncertainties in the parameters used in slope stability analyses

TABLE 2. Damage Probability Matrix from Permanent Deformation Analysis, Mode 1 (Example Case Study)

Mode 1 outcome (1)	Number of Equivalent Cycles 1–2 (2)	2–3 (3)	3–5 (4)	5–8 (5)	8–11 (6)
(*a*) $0.0g \le A < 0.1g$					
No/minor (*O*), < 2 ft	1.0	1.0	1.0	1.0	1.0
Heavy (*H*), 2–10 ft	0	0	0	0	0
Catastrophic (*C*), > 10 ft	0	0	0	0	0
(*b*) $0.1g \le A < 0.15g$					
No/minor (*O*), < 2 ft	0.998	0.991	0.952	0.801	0.703
Heavy (*H*), 2–10 ft	0.002	0.006	0.025	0.070	0.096
Catastrophic (*C*), > 10 ft	0	0.003	0.023	0.129	0.201
(*c*) $0.15g \le A < 0.2g$					
No/minor (*O*), < 2 ft	0.858	0.729	0.702	0.680	0.664
Heavy (*H*), 2–10 ft	0.094	0.149	0.126	0.100	0.084
Catastrophic (*C*), > 10 ft	0.048	0.122	0.172	0.220	0.252
(*d*) $0.2g \le A < 0.25g$					
No/minor (*O*), < 2 ft	0.735	0.702	0.677	0.656	0.642
Heavy (*H*), 2–10 ft	0.160	0.138	0.113	0.090	0.077
Catastrophic (*C*), > 10 ft	0.105	0.160	0.210	0.254	0.281
(*e*) $A \ge 0.25g$					
No/minor (*O*), < 2 ft	0.706	0.676	0.654	0.635	0.623
Heavy (*H*), 2–10 ft	0.153	0.126	0.101	0.081	0.068
Catastrophic (*C*), > 10 ft	0.141	0.198	0.245	0.284	0.309

Note: 1 ft = 0.305 m.

and thereby to estimate the probability of failure of a slope (A-Grivas and Asaoka 1982; Bergado and Anderson 1985; Catalan and Cornell 1976; Hadj-Hamou and Kavazanjian 1984; Vanmarcke 1977; Wu and Kraft 1970). Any suitable combination of these existing procedures can be used to estimate the probability of postearthquake failure of an earth dam.

It is desirable to present the probability results from mode 2 analysis in a matrix format similar to mode 1 as shown in Table 3. The numbers in the matrix correspond to the example presented subsequently. The matrix gives the probabilities of postearthquake failure for different accelerations and numbers of cycles, N_{eq}. Note that the matrix format facilitates combining the contributions of mode 1 and mode 2 to determine the overall seismic risk.

Combined Seismic Performance Analysis: Mode 1 and Mode 2

Tables 2 and 3 provide probabilities of damage and failure due to modes 1 and 2, respectively. The integration of the SPA matrix in Table 2 with the SHA results of Table 1 provides the seismic risk of damage associated with mode 1 to the dam. Integration of the SPA matrix in Table 3 with the

TABLE 3. Damage Probability Matrix from Postearthquake Stability Analysis, Mode 2 (Example Case Study)

Mode 2 outcome (1)	Number of Equivalent Cycles 1–2 (2)	2–3 (3)	3–5 (4)	5–8 (5)	8–11 (6)
	(*a*) $0.0g \le A < 0.1g$				
Survival (*S*)	1.0	1.0	1.0	1.0	1.0
Failure (*F*)	0	0	0	0	0
	(*b*) $0.1g \le A < 0.15g$				
Survival (*S*)	0.993	0.981	0.952	0.698	0.032
Failure (*F*)	0.007	0.019	0.048	0.302	0.968
	(*c*) $0.15g \le A < 0.2g$				
Survival (*S*)	0.849	0.032	0.032	0.032	0.032
Failure (*F*)	0.151	0.968	0.968	0.968	0.968
	(*d*) $0.2g \le A < 0.25g$				
Survival (*S*)	0.032	0.032	0.032	0.032	0.032
Failure (*F*)	0.968	0.968	0.968	0.968	0.968
	(*e*) $A \ge 0.25g$				
Survival (*S*)	0.032	0.032	0.032	0.032	0.032
Failure (*F*)	0.968	0.968	0.968	0.968	0.968

SHA results provides the risk of a postearthquake instability failure of the dam (mode 2). To calculate the overall seismic risk due to combined effects of modes 1 and 2, the damage probability matrices of Tables 2 and 3 can be combined first and then integrated with the SHA results.

Fig. 4 illustrates the possible outcomes of seismic performance of a dam

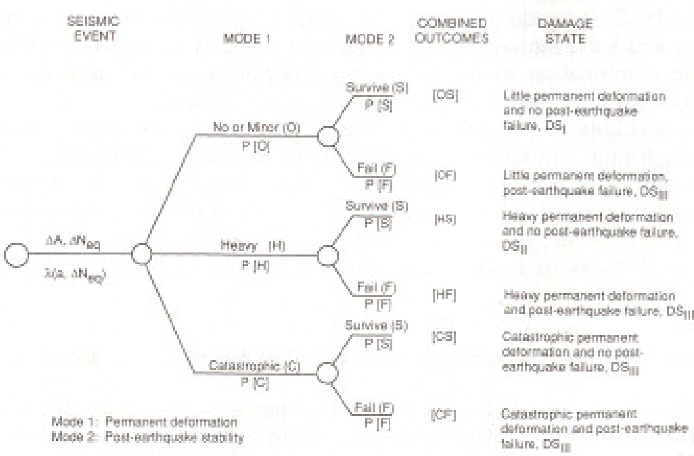

FIG. 4. Event Tree of Seismic Performance Analysis

TABLE 4. Damage Probability Matrix for Combined Modes (Example Case Study)

Mode 1 and 2 outcomes (1)	Number of Equivalent Cycles 1–2 (2)	2–3 (3)	3–5 (4)	5–8 (5)	8–11 (6)
	(*a*) $0.0g \le A < 0.1g$				
OS	1.0	1.0	1.0	1.0	1.0
HS	0	0	0	0	0
C/F	0	0	0	0	0
	(*b*) $0.1g \le A < 0.15g$				
OS	0.991	0.972	0.906	0.559	0.022
HS	0.002	0.006	0.024	0.049	0.003
C/F	0.007	0.022	0.070	0.392	0.975
	(*c*) $0.15g \le A < 0.2g$				
OS	0.728	0.023	0.022	0.022	0.021
HS	0.080	0.005	0.004	0.003	0.003
C/F	0.192	0.972	0.974	0.975	0.976
	(*d*) $0.2g \le A < 0.25g$				
OS	0.023	0.022	0.022	0.021	0.021
HS	0.005	0.004	0.004	0.003	0.002
C/F	0.972	0.974	0.974	0.976	0.977
	(*e*) $A \ge 0.25g$				
OS	0.023	0.022	0.021	0.020	0.020
HS	0.005	0.004	0.003	0.003	0.002
C/F	0.972	0.974	0.976	0.977	0.978

conditioned upon specified values of acceleration A, and number of equivalent cycles, N_{eq}. Note that the joint occurrence of any given pair of values of A and N_{eq} is in itself one of many possible outcomes, the probability of which is determined in the SHA. Given any pair of A and N_{eq}, the following outcomes are considered.

1. Possible outcomes for mode 1 are O, none to little permanent deformation, H, permanent deformation leading to heavy damage, and C, permanent deformation leading to catastrophic damage.
2. Possible outcomes for mode 2 are S, safe under postearthquake conditions (survival), and F, failure due to postearthquake conditions (failure).

Thus, there are six overall possible outcomes of the combination of modes 1 and 2.

To facilitate the evaluation of the performance of the embankment, the following damage states are defined in terms of the outcomes.

1. DS_I, outcome OS: little permanent deformation and no postearthquake failure.
2. DS_{II}, outcome HS: heavy permanent deformation but no postearthquake failure.

3. DS_{III}, outcome C/F: catastrophic permanent deformation, or postearthquake failure, or both.

If statistical independence is assumed between the mode 1 and the mode 2 outcomes, then the probability of occurrence for each damage state, $P(DS_i)$, can be expressed in terms of the probabilities of the possible outcomes from modes 1 and 2 as given by the following.

1. $P(DS_I) = P(O)P(S)$
2. $P(DS_{II}) = P(H)P(S)$
3. $P(DS_{III}) = P(O)P(F) + P(H)P(F) + P(C) = 1 - [P(O)P(S) + P(H)P(S)]$

Thus, the two damage probability matrices associated with mode 1 and mode 2 can be combined into a single matrix given in Table 4, which describes the overall results from the SPA.

Seismic Risk Analysis (SRA)

The calculation of the overall risk of damage or failure of a dam requires integration of the results of the SHA and the SPA. The use of matrices to display the results from the SHA (Table 1) and the SPA (Tables 2, 3, and 4) facilitates the numerical integration, by either a hand-held calculator or a computer, using the following expression:

$$\lambda(DS_i) = \sum_{\text{all}A} \sum_{\text{all}N_{eq}} P(DS_i|\Delta A, \Delta N_{eq}) \cdot \lambda(\Delta A, \Delta N_{eq}) \quad (2)$$

where $\lambda(DS_i)$ = expected annual number of events causing damage state DS_i; $\lambda(\Delta A, \Delta N_{eq})$ = annual number of events having acceleration range of ΔA and number of equivalent cycles of ΔN_{eq} (from the SHA matrix); $P(DS_i|\Delta A, \Delta N_{eq})$ = probability of damage state DS_i occurring, for each given pair of ΔA and ΔN_{eq} (from the SPA matrix); and i = damage states, I, II, and III. Eq. 2 gives the expected annual number of events associated with each damage state (DS).

The probability of at least one event causing a specified level of damage to a dam during the design life of the dam is calculated assuming a Poisson arrival process. For example, the probability of at least one catastrophic failure occurring in t years can be obtained from

$$P\ (\text{failure in } t \text{ years}) = 1 - e^{-\lambda(DS_{III})t} \quad (3)$$

where $\lambda(DS_{III})$ = the annual number of events causing catastrophic failure of the dam and is calculated using Eq. 2.

Example Case Study

To illustrate the various steps that are involved in the application of the seismic risk analysis, an embankment located near Boston was selected for investigation. The SHA for this example was discussed in an earlier section. Its results are shown in Table 1. This section presents the SPA of the embankment and the calculation of the overall seismic risk.

Fig. 2 shows a typical cross section of the embankment. The upstream

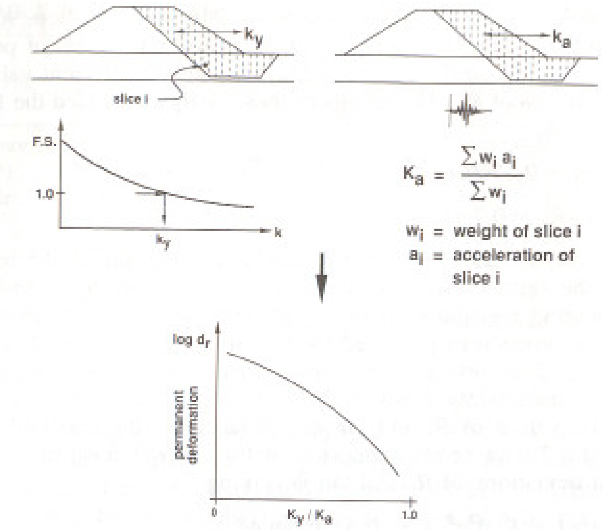

FIG. 5. Steps Involved in Calculation of Permanent Deformations

shell consists of medium-dense to dense sands. The downstream section consists primarily of loose to medium-dense sands and gravel with a loose sand layer near the base of the embankment. The embankment is founded on glacial till overlying bedrock. A field investigation indicated a layer of loose silty sand with low blow counts at about 40 ft (12.2 m) below the crest of the embankment, extending under the entire downstream shell of the embankment. The hatched region is a proposed fill that would increase the available freeboard of the embankment. For the purpose of illustration, the following analyses were performed for the embankment with the proposed fill assumed to have been already placed.

The risk of failure of the embankment is affected by both modes 1 and 2. Mode 2 failure is considered because the loose sand layer is susceptible to the development of excess pore water pressure and to loss of shear strength during the seismic excitations.

Fig. 5 indicates schematically the procedures followed to estimate the earthquake-induced permanent deformations of the embankment (mode 1). The average lateral earthquake-induced acceleration, K_a, for a critical sliding mass was estimated by performing dynamic response analysis of the embankment cross section. Specifically, a one-dimensional wave propagation analysis was performed for two idealized soil columns, one at the crest and the other near the toe of the embankment. The acceleration-time history applied at bedrock level was generated using the design response spectrum considered appropriate for the region. The average lateral acceleration, K_a, of the critical sliding mass shown in Fig. 2 was computed using the results of the simplified wave propagation analysis and the equation shown in Fig. 5.

The yield acceleration, K_y, of the critical wedge shown in Fig. 2 was estimated by performing slope stability analyses for different values of K_a.

The value of K_a giving a factor of safety of 1 is, by definition, K_y. In addition, the stability analyses were repeated for different levels of pore water pressure buildup, R_u, in the loose sand layer, yielding different values of K_y for different values of R_u. The results of these analyses yielded the following expression for K_y:

$$K_y = -0.091g + 0.571g(1 - R_u)\tan\phi \quad \text{for } R_u \leq 0.7 \quad (4a)$$

$$K_y = 0 \quad \text{for } R_u > 0.7 \quad (4b)$$

where $R_u = \Delta u/\bar{\sigma}_v$; Δu = the excess pore water pressure in the loose sand layer; $\bar{\sigma}_v$ = the vertical effective stress in the loose sand layer; and ϕ = the angle of shearing resistance of the sand. For each level of seismic input $(\Delta A, \Delta M)$, the procedures proposed by Yegian and Vitelli (1981) were used to estimate R_u. The yield acceleration K_y was estimated from R_u using Eq. 4*a*. However, uncertainty exists in the predicted values of K_y due to uncertainties in the values of R_u and $\tan\phi$. To calculate the standard deviation of K_y, σ_{K_y}, the Taylor series expansion of Eq. 4*a* was used to relate σ_{K_y} to the standard deviations of R_u and $\tan\phi$, giving

$$\sigma^2_{K_y} = [0.571g(1 - R_u)]^2\sigma^2_{\tan\phi} + (0.517g\tan\phi)^2\sigma^2_{Ru} \quad (5)$$

The standard deviation of R_u was estimated from the upper and lower bound curves developed by Yegian and Vitelli (1981). The coefficient of variation of $\tan\phi$ was taken as 0.15 (Harr 1977). Using Eq. 5, the standard deviation of K_y was estimated to be constant and equal to about 0.06g for all combinations of K_a and magnitude, M.

Permanent deformation of the mass, in this case due to sliding along the sand seam, occurs whenever K_a exceeds K_y. The probability that the permanent deformation accumulated during the earthquake will exceed any specified value can be estimated using the computer program NIMPED, or normalized plots, both of which are discussed in the companion paper. Its value depends on the mean values and standard deviations of K_y, and the predominant period, T, of the response of the sliding mass. The period, T, was estimated from the dynamic response analyses of the embankment, for different levels of ground acceleration. The standard deviation of T was estimated to be 0.8 s.

For this example, the following three damage states were defined for mode 1.

1. DS_{I} = no or minor damage = $[d_r \leq 2$ ft (0.61 m)].
2. DS_{II} = heavy damage = [2 ft (0.61 m) $< d_r \leq 10$ ft (3 m)].
3. DS_{III} = catastrophic damage = $[d_r > 10$ ft (3 m)].

The probabilities of occurrence of the damage states were calculated using the computer program NIMPED. Table 2 illustrates the resulting probabilities conditioned on the varying ranges of acceleration and equivalent number of cycles.

Recognizing that the loose saturated condition of the sand layer can lead to loss of strength during an earthquake and thus the posssibility of postearthquake instability, stability analyses of the embankment were made. Values of the factor of safety were calculated as a function of the pore pressure ratio, R_u, and the friction angle, ϕ, of the loose sand. The stability results

TABLE 5. Seismic Risk Results for Mode 1 (Example Case Study)

Outcome (1)	Permanent deformation (2)	Annual number of events ($\times 10^{-3}$) (3)	Annual probability (4)	Probability in 50 yr (%) (5)
No or minor (*O*)	Less than 2 ft	221.903	0.9996	98.14
Heavy (*H*)	2 to 10 ft	0.165	0.165×10^{-3}	0.81
Catastrophic (*C*)	Greater than 10 ft	0.211	0.21×10^{-3}	1.05

Note: 1 ft = 0.305 m.

suggest the following relationship between the postearthquake factor of safety, *F.S.*, and the parameters R_u and ϕ:

$$F.S. = 0.66 + 2(1 - R_u)\tan\phi \quad (6)$$

Eq. 6 can be used to estimate the mean value of *F.S.* from the mean values of R_u and ϕ. The calculation of the probability of postearthquake slope failure requires the estimation of the standard deviation of *F.S.* Using the Taylor series expansion for Eq. 6, the standard deviation of *F.S.* can be obtained from the resulting expression

$$\sigma^2_{F.S.} = [2(1 - R_u)]^2\sigma^2_{\tan\phi} + (2\tan\phi)^2\sigma^2_{R_u} \quad (7)$$

Using the estimated mean value of *F.S.* and $\sigma_{F.S.}$, for each seismic event, the probability of failure was calculated assuming the factor of safety to be a normally distributed random variable. Table 3 provides the resulting damage probability matrix for mode 2.

The damage probabilities from mode 1 and mode 2 can be combined as described earlier. Table 4 provides the combined damage probabilities assuming that the catastrophic damage from mode 1 and failure from mode 2 are statistically independent and have the same loss function, thus limiting the number of combined damage states to three.

The results shown in the SHA (Table 1) and SPA (Tables 2, 3, and 4) matrices were integrated using Eq. 2 to provide the annual number of seismic events that would cause a specified damage state.

The resulting annual number of occurrences of damage states I, II, and III are presented for mode 1 in Table 5 and for mode 2 in Table 6. Similar results are illustrated in Table 7, considering the combined effect of mode 1 and mode 2. The annual probability of at least one occurrence of a damage state can be obtained assuming the Poisson arrival process. Thus

$$P(DS_{\text{III}}) = 1 - e^{-\lambda(DS_{\text{III}})} = 1 - e^{-(0.001060)} = 1.060 \times 10^{-3} \quad (8)$$

$$P(DS_{\text{II}}) = 1 - e^{-[\lambda(DS_{\text{II}})+\lambda(DS_{\text{III}})]} - P(DS_{\text{III}}) = 1 - e^{-(0.001104)} - 1.060 \times 10^{-3}$$

TABLE 6. Seismic Risk Results for Mode 2 (Example Case Study)

Outcome (1)	Annual number of events ($\times 10^{-3}$) (2)	Annual probability of exceedence (3)	Probability in 50 yr (%) (4)
Survival (*S*)	221.259	0.999	95.0
Failure (*F*)	1.020	0.001	5.0

TABLE 7. Seismic Risk Results for Combined Modes (Example Case Study)

Combined outcome (1)	Damage state (2)	Annual number of events (× 10^{-3}) (3)	Annual probability (4)	Probability in 50 yr (%) (5)
OS	No or minor damage	221.174	0.9989	94.63
HS	Heavy damage	0.044	0.043×10^{-3}	0.21
C/F	Failure	1.060	1.060×10^{-3}	5.16

$$= 0.043 \times 10^{-3} \quad (9)$$

$$P(DS_I) = 1 - [P(DS_{II}) + P(DS_{III})] = 0.9989 \quad (10)$$

The seismic risk results shown in Tables 5 and 6 indicate that the primary contributions to the overall risk of catastrophic failure of the example embankment is from the mode 2 type of failure. Therefore, for the example case study, all assumptions and parameters related to the postearthquake instability analyses are comparatively more important than those related to permanent deformations.

Summary and Conclusions

There are significant uncertainties involved in the evaluation of the seismic safety of an earth dam. A methodology is presented for seismic risk analysis for earth dams that accounts for uncertainties in the seismological and geotechnical parameters and in the procedures used for analysis of earth dams. The application of risk-based methodology to the seismic evaluation of an earth dam can provide a rational basis for the selection of input parameters and for the design decision analysis. In addition, calculation of relative risk can help identify the most important parameters, assumptions, and safety criteria affecting the evaluation of the dam and can avoid compounding of conservatism.

Acknowledgments

The research described in this report was sponsored by the National Science Foundation through Grant No. DFR-84-12124 for research on integrated seismic risk analysis for earth dams. The writers gratefully acknowledge this support.

Appendix I. References

A-Grivas, D., and Asaoka, A. (1982). "Slope safety prediction under static and seismic loads." *J. Geotech. Engrg. Div.*, ASCE, 108(5), 713–729.

Ambraseys, N. N., and Menu, J. M. (1988). "Earthquake-induced ground displacements." *Earthquake Engrg. and Struct. Dynamics*, 16, 985–1006.

Bergado, D. T., and Anderson, L. R. (1985). "Stochastic analysis of pore pressure uncertainty for the probabilistic assessment of the safety of earth slopes." *Soils and Found.*, 25 (Jun.), 87–105.

Castro, G. (1976). "Comments on seismic stability evaluation of embankment dams." *Proc. Engrg. Found. Conf. on Evaluation of Dam Safety*, Pacific Grove, Calif., Earthquake Engrg. Res. Inst., 377–390.

Catalan, J. M., and Cornell, C. A. (1976). "Earth slope reliability by a level-crossing method." *J. Geotech. Engrg. Div.*, 102(6), 591–604.

Chaney, R. C. (1979). "Earthquake induced deformations in earth dams." *Proc. 2nd U.S. Nat. Conf. on Earthquake Engrg.*, Stanford Univ., Stanford, Calif., Earthquake Engrg. Res. Inst., 633–642.

Cornell, C. A. (1968). "Engineering seismic risk analysis." *Bull. Seismological Soc. of Amer.*, 58(4), 1583–1606.

Cornell, C. A., and Merz, H. A. (1974). "A seismic risk analysis of Boston." *Seismic Design Decision Analysis*, Report No. 11, Massachusetts Inst. of Tech., Cambridge, Mass.

Dobry, R., Ladd, R. S., Yokel, F. Y., and Chung, R. M. (1982). "Prediction of pore water pressure buildup and liquefaction of sands during earthquakes by the cyclic strain method." *Bldg. Sci. Series 138*, Nat. Bureau of Standards, U.S. Dept. of Commerce, Washington, D.C.

Franklin, A. G., and Chang, F. K. (1977). "Permanent displacements of earth embankments by Newmark sliding block analysis." *Earthquake Resistance of Earth and Rock-Fill Dams*, Report 5, Misc. Paper No. S-71-17, U.S. Army Engr. Waterways Experiment Station, Vicksburg, Miss.

Gazetas, G. (1987). "Seismic response of earth dams: Some recent developments." *Soil Dynamics and Earthquake Engrg.*, 6(1), 2–47.

Hadj-Hamou, T., and Kavazanjian, E. (1984). "Probabilistic seismic stability of a cohesionless slope of limited extent." *Proc. 8th World Conf. on Earthquake Engrg.*, San Francisco, Calif., Earthquake Engrg. Res. Inst., 3, 429–436.

Harr, M. (1977). *Mechanics of particulate media*. McGraw-Hill Book Co., New York, N.Y.

Idriss, I. M. (1985). "Evaluating seismic risk in engineering practice." *Proc. 11th Int. Conf. on Soil Mech. and Foundation Engrg.*, San Francisco, Calif., Int. Society for Soil Mech. and Foundation Engrg., 1, 255–320.

Ishihara, K. (1984). "Post-earthquake failure of a tailing dam due to liquefaction of the pond deposit." *Proc. Int. Conf. on Case Histories in Geotech. Engrg.*, Univ. of Missouri, Rolla, Mo., 3, 1129–1143.

Kikusawa, M., and Hasegawa, T. (1985). "Analysis of model embankment dam by shaking table test." *Soils and Found.*, 25(1), 1–14.

Klohn, E. J., Maartman, C. M., Lo, R. C. Y., and Finn, W. D. L. (1978). "Simplified seismic analysis for tailing dams." *Proc. Specialty Conf. on Earthquake Engrg. and Soil Dynamics*, ASCE, New York, N.Y., 540–556.

Lin, J. S., and Whitman, R. V. (1986). "Earthquake induced displacements of sliding blocks." *J. Geotech. Engrg. Div.*, ASCE, 112(1), 44–59.

Makdisi, F. I., and Seed, H. B. (1978). "Simplified procedure for estimating dam and embankment earthquake-induced deformations." *J. Geotech. Engrg. Div.*, ASCE, 104(7), 849–867.

Mohamad, R., Dakoulas, P., Gazetas, G., and Dobry, R. (1985). "Liquefaction flow failure evaluation of earth dams." *Proc. Eleventh Int. Conf. on Soil Mech. and Foundation Engrg.*, San Francisco, Calif., Int. Society for Soil Mech. and Foundation Engrg., 1865–1868.

Newmark, N. M. (1965). "Effects of earthquakes on dams and embankments." *Geotechnique*, 15(2), 139–160.

Peck, R. B. (1977). "Pitfalls of overconservatism in geotechnical engineering." *Civil Engrg.*, ASCE, 47(2), 62–66.

Ramanujam, N., Holish, L. L., and Chen, W. W. H. (1978). "Post earthquake stability analysis of earth dams." *Proc. Specialty Conf. on Earthquake Engrg. and Soil Dynamics*, ASCE, New York, N.Y., 762–776.

Reiter, L., and Jackson, R. E. (1983). "Seismic hazard review for the systematic evaluation program—A use of probability in decision making." NUREG-0967, U.S. Nuclear Regulatory Commission, Washington, D.C.

Schumacker, B., and Whitman, R. V. (1978). "Preliminary seismic risk study of Orinoco delta and computer program for seismic risk." *Res. Report No. R78-39*, Massachusetts Inst. of Tech., Cambridge, Mass.

Seed, H. B. (1987). "Design problems in soil liquefaction. *J. Geotech. Engrg.*, ASCE, 113(8), 827–845.
Seed, H. B., Idriss, I. M., and Arango, I. (1983). "Evaluation of liquefaction potential using field performance data." *J. Geotech. Engrg.*, ASCE, 109(3), 458–482.
Seed, H. B., Lee, K. L., Idriss, I. M., and Makdisi, F. I. (1975). "The slides in the San Fernando dams during the earthquake of February 9, 1971." *J. Geotech. Engrg. Div.*, ASCE, 101(7), 651–688.
Taniguchi, E., Whitman, R. V., and Marr, W. A. (1983). "Prediction of earthquake-induced deformation of earth dams." *Soils and Found.*, 23(4), 126–132.
Tong, W. H., Schumacker, B., Cornell, C. A., and Whitman, R. V. (1975). "Seismic hazard maps for Massachusetts." *Seismic Design Decision Analysis Series, Internal Report No. 52*, Dept. of Civ. Engrg., Massachusetts Inst. of Tech., Cambridge, Mass.
Vanmarcke, E. H. (1977). "Reliability of earth slopes." *J. Geotech. Engrg. Div.*, ASCE, 103(11), 1247–1265.
Whitman, R. V. (1984). "Evaluating calculated risk in geotechnical engineering." *J. Geotech. Engrg. Div.*, ASCE, 110(2), 145–188.
Wu, T. H., and Kraft, L. M. (1970). "Safety analysis of slopes." *J. Soil Mech. and Found. Div.*, ASCE, 96(2), 609–630.
Yegian, M. K. (1979). "Probabilistic seismic hazard analysis." *State-of-the-Art for Assessing Earthquake Hazards in the United States*, Report 13, U.S. Army Engr. Waterways Experiment Station, Vicksburg, Miss.
Yegian, M. K. (1984). "Probabilistic seismic hazard analysis for pore pressure build-up in sands." *Proc. 8th World Conf. on Earthquake Engrg.*, San Francisco, Calif., Earthquake Engrg. Res. Inst., 3, 167–174.
Yegian, M. K., Marciano, E. A., and Ghahraman, V. G. (1988). "Integrated seismic risk analysis for earth dams." *Res. Report No. CE88-15*, Dept. of Civ. Engrg., Northeastern Univ., Boston, Mass.
Yegian, M. K., Marciano, E. A., and Ghahraman, V. G. (1991). "Earthquake-induced permanent deformations: Probabilistic approach." *J. Geotech. Engrg.*, ASCE, 117(1), 35–50.
Yegian, M. K., and Vitelli, B. M. (1981). "Probabilistic analysis for liquefaction." *Res. Report No. CE-81-1*, Civ. Engrg. Dept., Northeastern Univ., Boston, Mass.

Appendix II. Notation

The following symbols are used in this paper:

A = peak ground acceleration;
DS_i = damage state i;
d_r = specified permanent deformation;
$F.S.$ = factor of safety in post earthquake slope stability analysis;
g = gravitational constant;
K_a = average acceleration of a critical sliding mass;
K_y = yield acceleration of critical sliding mass;
M = Richter magnitude;
m_{max} = maximum credible magnitude;
m_0 = minimum magnitude of interest;
N_{eq} = number of equivalent uniform cycles;
$P(F)$ = probability of failure due to post-earthquake conditions;
$P(H)$ = probability of permanent deformation leading to heavy damage;
$P(O)$ = probability of none to little permanent deformation;
$P(S)$ = probability of survival under post-earthquake conditions;
R_u = pore pressure ratio;

SHA = seismic hazard analysis;
SPA = seismic performance analysis;
SRA = seismic risk analysis;
T = period of motion of a critical sliding mass;
β = magnitude-frequency parameter defined by Cornell (1968);
Δu = excess pore water pressure;
$\lambda(A > a)$ = annual number of events causing acceleration A exceeding a specified value a;
$\lambda(a, \Delta M)$ = annual number of events causing acceleration to exceed a and having magnitudes greater than m_0 but less than m_i;
$\lambda(DS_i)$ = annual number of events causing damage state, DS_i;
σ_T = standard deviation of T;
$\bar{\sigma}_v$ = vertical effective stress; and
ϕ = angle of shearing resistance.